國立成功大學機械工程學系

機械振動學(Mechanical Vibrations)

Spring Term 2023

QUIZ I

April 18 2020 (Tuesday)
3:10 – 5:10 PM

RM. 91302

Note:
Problem I: Close Book/Close Notes
Rest Problems: Close Book but a A4 sheet of notes is permissible

先做第一題. 第一題交卷後, 可拿出預先準備的 Note Sheet, 做其他的題目.

共 6 大題 + 1 Bonus

Total 113 Points + 10 Bonus Points

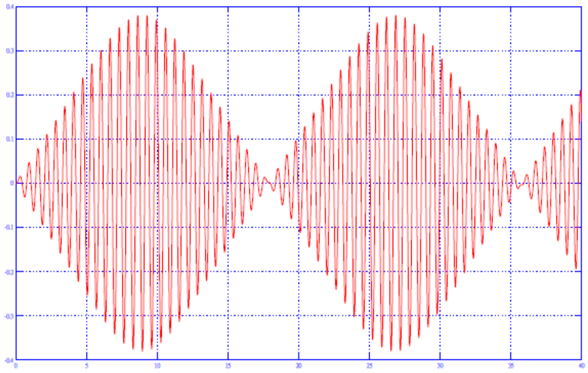
Part I. Closed books/notes (8 Problems, 40 Pts, 5 Pts/each)

Problem 1. Please briefly answer the following questions (Closed books/notes)

A. Please define the term "resonance". How to find resonance frequency for overdamped system?

B. Please state clearly the procedure of Rayleigh's method for determining the natural frequency.

C. In class, we use Matlab/Simulink to show that for a vibration system with nonlinear spring excited at its linear natural frequencies, the vibration amplitude would initially grow but then decrease, then grow, then decrease, and so on. Please use your words to explain.

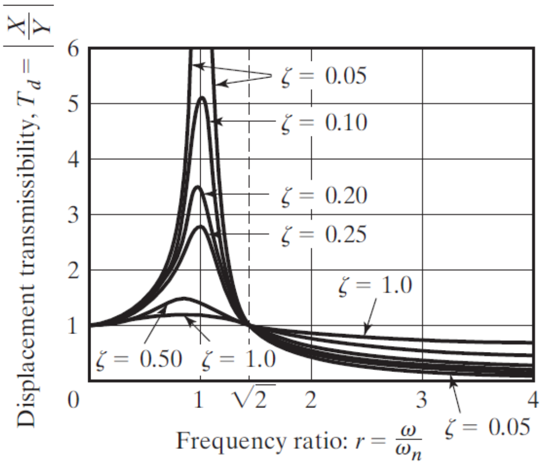


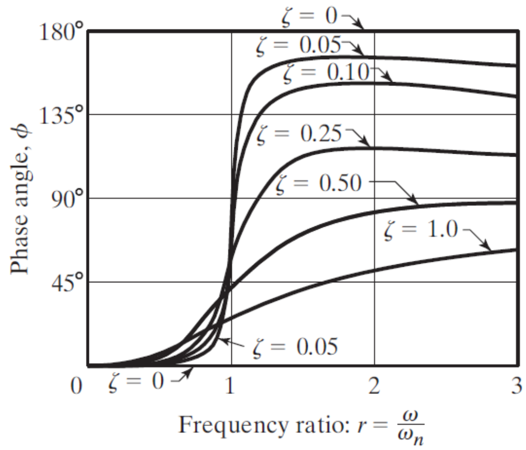
D. For a SDOF vibrating system with coulomb damping under sinusoidal excitation (force amplitude F_0, frequency ω), its amplitude X is expressed as Eq.(3.93). I.e.,

$$X = \frac{F_0}{k}\left[\frac{1 - \left(\frac{4\mu N}{\pi F_0}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}\right]^{1/2} \tag{3.93}$$

Where N, μ, k are the normal force, friction coefficient, and spring constant, respectively. ω_n is the system natural frequency. Obviously, at resonance, the displacement amplitude is unbounded. Please use physics to explain it.

E. Consider the following figure, please define the term *displacement transmissibility*. In addition, for a SDOF system with m=1 Kg, c = 7 N.s/m, k=1000N/m, subjected to an external vibration with ω=25 rad/s, please estimate its transmissibility. R~0.8, eta~ 0.1, TR~ 3.5, angle~35deg.



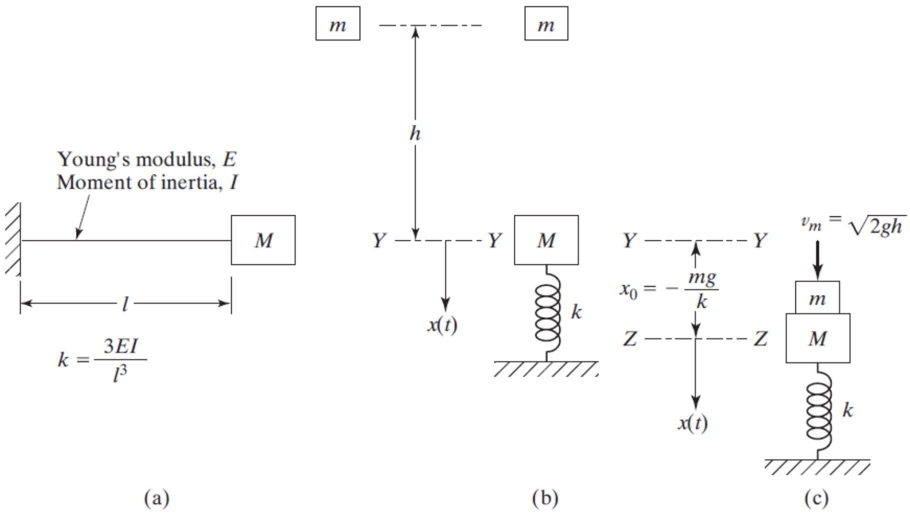


F. Please define the *loss coefficient* (or *loss factor)* and *quality factor* of a vibration system and state clearly their physical meanings.

G. In class, we have represented the vibration problem using phasor representation. Please sketch the phasor diagram for the situation when ω/ω_n >>1. Please also use the diagram to explain that the phase lag is near 180°.

H. 底下是課本例題 2.2. 在此並非要請各位解出該題目. 而是要請各位說明如何決定該振動問題之 initial conditions (i.e., initial velocity 與 initial displacement). 請試著用物理(及配合數學)方式說明. 讓我知道你懂.

A cantilever beam carries a mass M at the free end as shown in Fig. 2.11(a). A mass m falls from a height h onto the mass M and adheres to it without rebounding. Determine the resulting transverse vibration of the beam.



Part II. A sheet of notes is permissible (5 Problems + 1 Bonus, 63 + 10 Pts)

Problem 2. *(Experimental Extraction of System Parameter)* ***(10 Pts)***

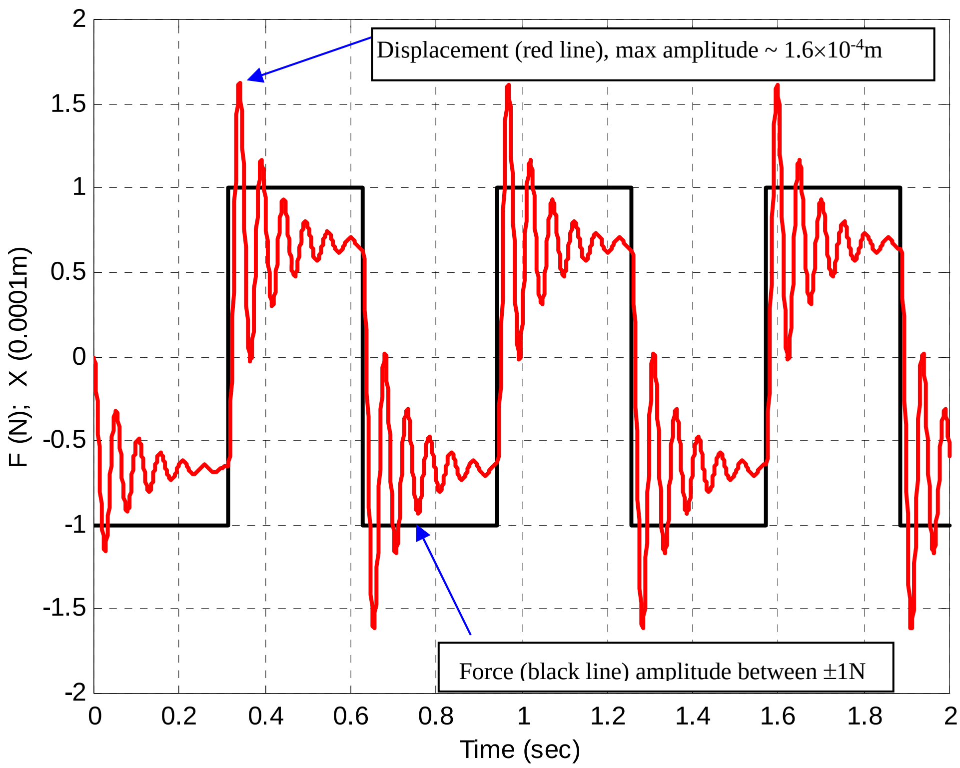
Consider the following figures, a SDOF free vibration system subjected to a square wave excitation (with force amplitude between ±1N). The equation of motion can be expressed as

$$m\ddot{x} + b\dot{x} + kx = F(t),$$

where F(t) is the excitation and is shown below. You are asked to determine the system parameters (i.e., m, b, and k) based on the response and the input.

(a) Without any calculations, please tell us how to find m, b, k. (5 Pts)

(b) Please use the method you answered in (a) to calculated m, b, k. (5 Pts)



SOL: at steady state, F/x = K = 1/ 0.00006 ~ 1.67 x 10^4 N/m

Estimated damped natural period ~ 0.21/4 = 0.0525 s → damped natural frequency ~ 120 rad/s

By Log decrement method, ln(x1/x2) ~ ln(1.6x10^{-4}/1.15x10^{-4})=0.33 --> damping ratio ~ 0.05. this implies that the natural frequency ~ 120 rad/s.

M = K/ωn^2 → M ~ 1.16 Kg

b = 0.05Cc = 0.05 x 2*m*ωn = 13.9 N.s/m.

Problem 3. Vehicle Suspension Vibration **(10 Pts)**

The spring of an automobile trailer is compressed **10.16cm** under its weight. The trailer is traveling over a road with a profile approximated by a sine wave of amplitude **7.62cm** and wavelength of **14.63m**.

(a) Please write the equation of motion.

(b) Please find the critical speed for exciting the resonance.

(c) What will be the amplitude of vibration at **64.4 km/h**? (assuming no damping)

SOL:

Assume the road profile = y(t). The equation of motion can be written as

$$m\ddot{x} = -k(x-y)$$

or

$$m\ddot{x} + kx = ky$$

Let $y(t) = Y\sin\frac{2\pi Vt}{\lambda}$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = 1.563Hz$$

or

$$\omega_n = 2\pi f_n = 9.82 rad/s$$

Speed to excite resonance: $\frac{2\pi V_c}{\lambda} = \omega_n$ → V_c = 22.9 m/s=82.4 km/h (λ=14.63m)

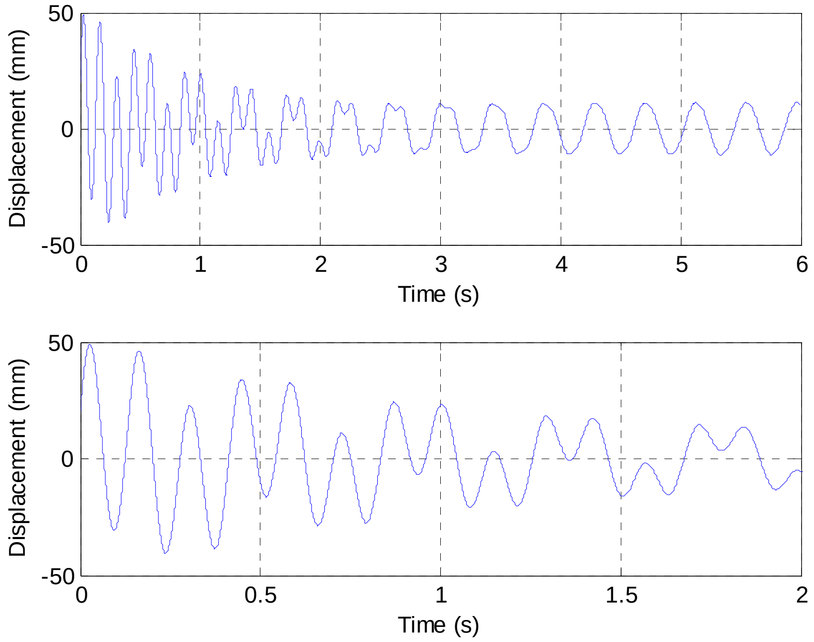
$$V = 64.4 km/hr = 17.9 m/s$$

Exciting frequency $\omega = 2\pi\frac{V}{\lambda} = 7.68 rad/s$

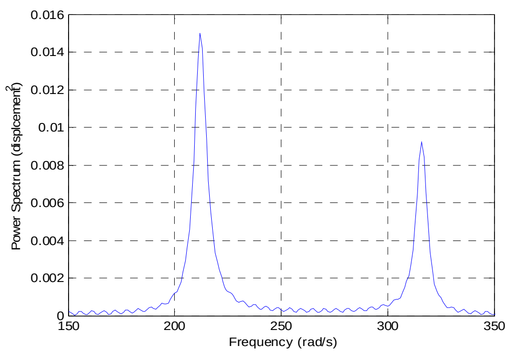
$$(\frac{\omega}{\omega_n})^2 = (\frac{7.68}{9.82})^2 = 0.612 \quad \rightarrow \quad X = \frac{7.62cm}{1-(\frac{\omega}{\omega_n})^2} = 19.62cm$$

Problem 4. Experimental Vibrations **(21 Pts)**

a. Consider a SDOF system subjected to a sinusoidal excitation and the response is shown below. Please estimate the frequency of the exciting signal and the system natural frequency. ANS: $\ddot{x} + 2\dot{x} + 2000x = 10\sin(15t)$. Driving = 15 rad/s (2.39 Hz), Natural freq = 44.7 rad/s   7.12 Hz)

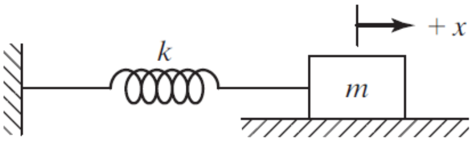


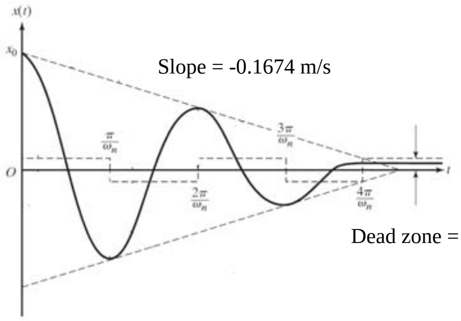
b. A typical portion of the power spectrum of a structural test is shown in below. Note: this is a power spectrum plot, which means that the y-axis (power spectrum) value is proportional to the square of vibration amplitude. Please estimate the damping ratios associated with the two modes. Also, please briefly plot the free vibration response of the system with an initial displacement of 10 mm. Please clearly indicate the signal decay and showing the vibration periods.



c. As shown in the following figures, a spring-mass system moves back and forth on a **horizontal** friction surface. Assume the spring constant is 1000 N/m and mass is 2 Kg. The response is also shown below. It moves in back and forth with decayed amplitude and a final dead distance. Please estimate the dynamic and static friction coefficients.

SOL: slope $= -2\mu_k N\omega_n / \pi k = -0.279\mu_k$. Dead zone $=\mu_s N/k = 0.0196\mu_s$. → static coef = 0.8; dynamic coef = 0.6. N = 19.6 N, m = 2 Kg, wn= 22.36 rad/s.





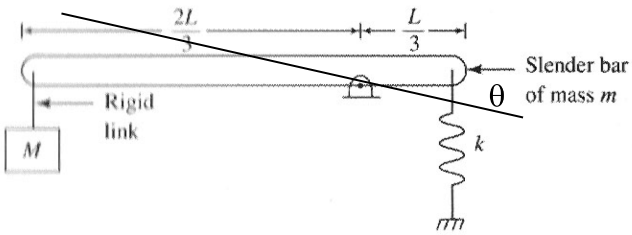
Problem 5. Vibration Equation **(16 Pts)**

Consider the system shown in the following figure.

(a) Is the system a conservative system? Why? (4 Pts)

(b) Please use θ as the variable and express the kinetic energy and potential energy of this system (6 pts)

(c) Please find the equation of motion and its natural frequency (6 pts)



Let θ be the clockwise angular displacement of the bar from the system's equilibrium position. Assuming small θ, the potential energy of the system at an arbitrary instant is

$$V = \frac{1}{2}k\left(\frac{L}{3}\theta\right)^2 = \frac{1}{2}\left(\frac{1}{9}kL^2\right)\theta^2 \rightarrow k_{t_{eq}} = \frac{1}{9}kL^2$$

The kinetic energy of the system at an arbitrary instant is

$$T = \frac{1}{2}\frac{1}{12}mL^2\dot{\theta}^2 + \frac{1}{2}m(\frac{1}{6}L\dot{\theta})^2 + \frac{1}{2}M(\frac{2}{3}L\dot{\theta})^2 = \frac{1}{2}(\frac{1}{9}mL^2 + \frac{4}{9}ML^2)\dot{\theta}^2 \rightarrow I_{eq} = \frac{1}{9}mL^2 + \frac{4}{9}ML^2$$

The governing differential equation is

$$(\frac{1}{9}mL^2 + \frac{4}{9}ML^2)\ddot{\theta} + \frac{1}{9}kL^2\theta = 0$$

$$\ddot{\theta} + \frac{k}{m + 4M}\theta = 0$$

from which the natural frequency is determined as

$$\omega_n = \sqrt{\frac{k}{m + 4M}}$$

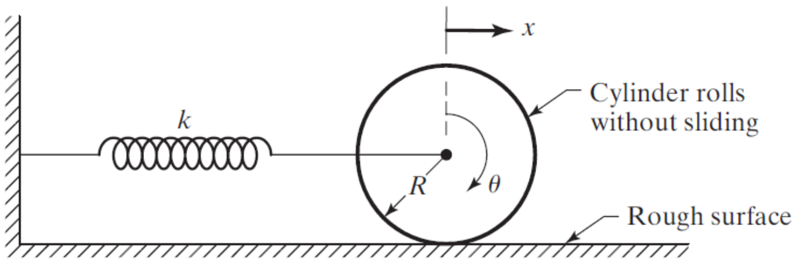
Problem 6. Rayleigh Method **(16 Pts)**

A cylinder of mass ***m*** and mass moment of inertia ***J*** is connected to a spring of stiffness ***k*** and rolls on a rough surface as shown below. If the translational and angular displacements of the cylinder are ***x*** and $\boldsymbol{\theta}$ from its equilibrium position, please using **ENERGY METHOD** to determine the equation of motion of the system for small displacements

(a) in terms of ***x***

(b) in term of $\boldsymbol{\theta}$

You can choose either in terms of ***x*** or in term of $\boldsymbol{\theta}$ **but not for both.**



Assume: No sliding of the cylinder.

Kinetic energy of the cylinder (T) = sum of translational and rotational kinetic energies

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 \qquad (E_1)$$

Since the cylinder rolls without sliding,

$$x = \theta R \quad \text{or} \quad \theta = \frac{x}{R} \qquad (E_2)$$

Using Eq. (E_2), the kinetic energy can be expressed as

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \cdot \frac{\dot{x}^2}{R^2} = \frac{1}{2}\left(m + \frac{J}{R^2}\right) \cdot \dot{x}^2 \qquad (E_3)$$

$$= \frac{1}{2} m \dot{\theta}^2 R^2 + \frac{1}{2} J \dot{\theta}^2 = \frac{1}{2}\left(mR^2 + J\right)\dot{\theta}^2 \qquad (E_4)$$

The potential (or strain) energy, U, due to the deflection of the spring is given by

$$U = \frac{1}{2} k x^2 \qquad (E_5)$$

$$= \frac{1}{2} k R^2 \theta^2 \qquad (E_6)$$

Total energy is constant since the damping is absent.

$$\therefore \quad T + U = c = \text{constant} \qquad (E_7)$$

Using Eqs. (E_3) and (E_5) in Eq. (E_7), we obtain

$$\frac{1}{2}\left(m+\frac{J}{R^2}\right)\dot{x}^2+\frac{1}{2}kx^2=c \qquad (E_8)$$

Differentiating Eq. (E_8) w.r.t. time gives

$$\frac{1}{2}\left(m+\frac{J}{R^2}\right)(2\dot{x})\ddot{x}+\frac{1}{2}k(2x\dot{x})=0$$

or

$$\left[\left(m+\frac{J}{R^2}\right)\ddot{x}+kx\right]\dot{x}=0 \qquad (E_9)$$

Since $\dot{x}\neq 0$ for all t,

$$\left(m+\frac{J}{R^2}\right)\ddot{x}+xk=0 \qquad (E_{10})$$

The natural frequency of vibration, from Eq. (E_{10}), is given by

$$\omega_n=\sqrt{\frac{k}{\left(m+\frac{J}{R^2}\right)}} \qquad (E_{11})$$

Since the mass moment of inertia of a cylinder can be expressed as

$$J=\frac{1}{2}mR^2 \qquad (E_{12})$$

Eqs. (E_{10}) and (E_{11}) become

$$\frac{3}{2}m\ddot{x}+kx=0 \qquad (E_{13})$$

$$\omega_n=\sqrt{\frac{2k}{3m}} \qquad (E_{14})$$

Using Eqs. (E_4) and (E_6), the total energy of the system can be expressed as

$$\frac{1}{2}(mR^2+J)\dot{\theta}^2+\frac{1}{2}kR^2\theta^2=c=\text{constant} \qquad (E_{15})$$

Differention of Eq. (E_{15}) with respect to time gives

$$\frac{1}{2}(mR^2+J)(2\dot{\theta}\ddot{\theta})+\frac{1}{2}kR^2(2\theta\dot{\theta})=0 \qquad (E_{16})$$

$$\left[(mR^2+J)\ddot{\theta}+kR^2\theta\right]\dot{\theta}=0 \qquad (E_{17})$$

Since $\dot{\theta}\neq 0$ for all t,

$$(mR^2 + J)\ddot{\theta} + kR^2\theta = 0 \quad (E_{18})$$

The natural frequency of vibration, from Eq. (E_{18}), is given by

$$\omega_n = \sqrt{\frac{kR^2}{mR^2 + J}} \quad (E_{19})$$

Using Eq. (E_{12}), Eqs. (E_{18}) and (E_{19}) become

$$\frac{3}{2} mR^2 \ddot{\theta} + kR^2\theta = 0 \quad (E_{20})$$

$$\omega_n = \sqrt{\frac{kR^2}{\frac{3}{2} mR^2}} = \sqrt{\frac{2k}{3m}} \quad (E_{21})$$

It can be seen that the two equations of motion, Eqs. (E_{10}) and (E_{18}), lead to the same natural frequency ω_n as shown in Eqs. (E_{14}) and (E_{21}).

Bonus:

Please answer the following questions briefly

(a) What is a skyhook damper? How to implement it and what is its major advantages?

(b) Please use your own words to define the term "equivalent mass" in vibration model.