國立成功大學機械工程學系

機械振動學(Mechanical Vibrations)

Spring Term 2022

QUIZ II

June 01 2022 (Wednesday)

6:30 – 8:30 PM (可延長至 21:00)

RM. 91204

Note:

Problem I: Close Book/Close Notes

Rest Problems: Close Book but a A4 sheet of notes is permissible (請不要在 handout 上面黏貼其它的資料)

先做第一題. 第一題交卷後, 可拿出預先準備的 Notes Sheet, 做其他的題目.

共 8 大題. 本次題目 (含此頁) 計 6 頁

Total 134 Points

Part I. Closed books/notes (8 Problems, 40 Pts, 5 Pts/each)

Problem 1. Please briefly answer the following questions (Closed books/notes)

A. Please use your words to define the response spectrum of a vibration system and tell us its engineering applications

B. For a SDOF system subjected to a general non-periodic loading shown below, the general solution can be expressed as:

$$x(t) = \frac{1}{m\omega_d}\int_0^t F(\tau)\, e^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t-\tau)\, d\tau$$

Please try your best to tell us the physics/math of the above equation and to explain why the equation is the general solution of the vibration problem.

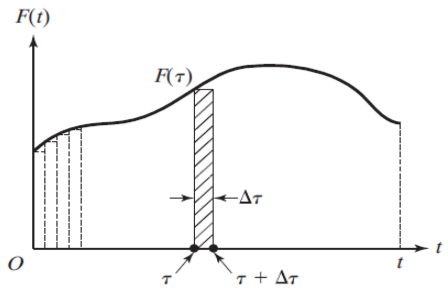


FIGURE 4.9 An arbitrary (nonperiodic) forcing function.

C. In chapter 5, we have seen that a multi-degree of freedom vibration system is usually coupled. There are several types of couplings such as static (i.e., spring) and dynamic (i.e., mass) coupling. Please define these two types of couplings. Also, please try your best to tell us the nature of these couplings. (即: 為何系統會出現 static /dynamic couplings).

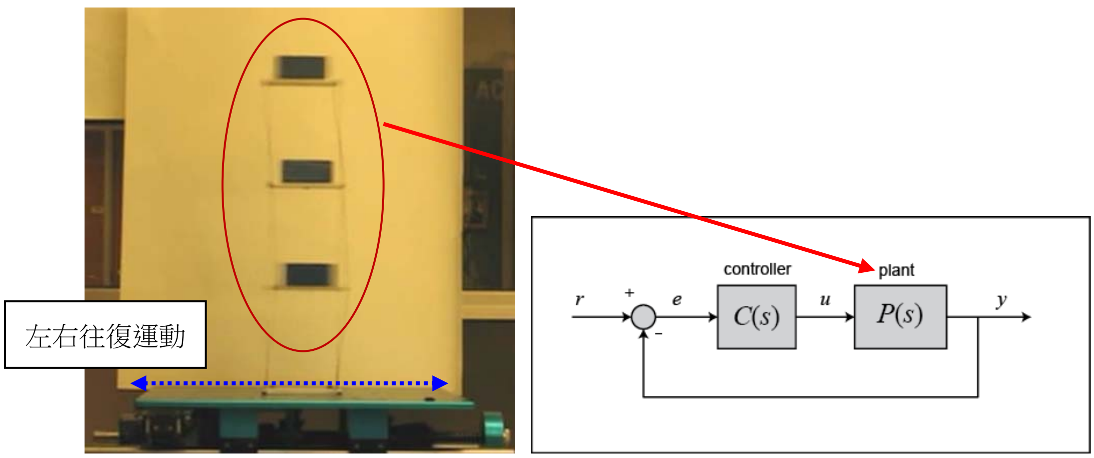
D. In MDOF systems, why [K] matrix is symmetric ?

E. What is principal coordinates and mode shapes? Please try to explain it by mathematics and physics/engineering

F. What happen if a vibration system contains eigenvalues of value = 0? And what happen if the system has repeated eigenvalues?

G. Please use your words to explain the reciprocal theorem and tell us the possible applications.

H. 底下是一個典型的結構. 假設你被要求設計一控制器, 藉以壓制該結構之側向振動. 首先你必須自己設計實驗去求取該 plant 之 振動參數乃至於整個 transfer function. 而 transfer function 又可以分解成 gain, zero, pole 等三要素. 請大略說明你如何求取該系統之 poles 與 zeros.



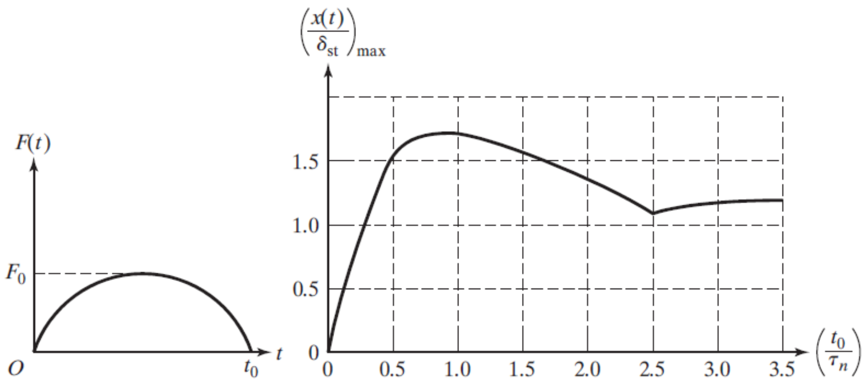
Part II. A sheet of notes is permissible (7 Problems, 84 Pts)

Problem 2.　Construction of Response Spectrum (10 Pts)

考慮一個 forced SDOF mass-spring system (沒有 damper).

$$m\ddot{x} + kx = F(t) = \begin{cases} F_0 \sin \omega t, & 0 \le t \le t_0 \\ 0, & t > t_0 \end{cases} \quad 其中 \quad \omega = \frac{\pi}{t_0}$$

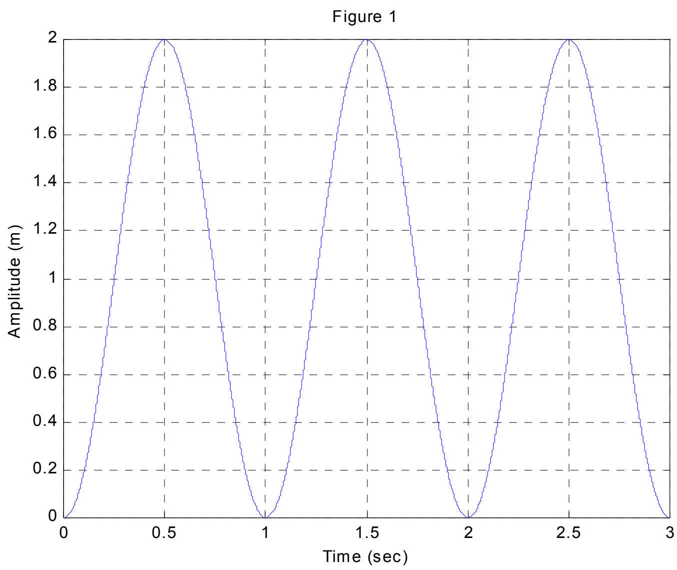
其 forcing function 如圖下左所示，而其 response spectrum 如圖下右所示. 請告訴我們如何獲取該 response spectrum. 你不需要很複雜詳細的數學，可以以數學或模擬實驗操作的方式說明. 評分標準為讓我們認為你的確是懂，而不是拋出一堆數學煙幕彈.

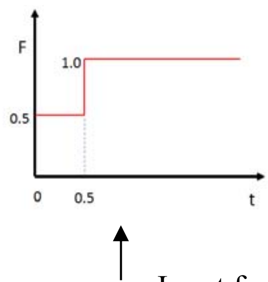


(a)　　(b)

Problem 3.　Input Shaping　(10 Pts)

The following figure shows the unit-step response of a SDOF vibration system. Please briefly predict the response of this system subject to a input shown in the figure.





Input force

Unit step response

Problem 4.　M-DOF system (I): System Modeling　(10 Pts)

注意：底下題目不是要你去解它，而是請你說明其所附的解答是否有誤．並說明之

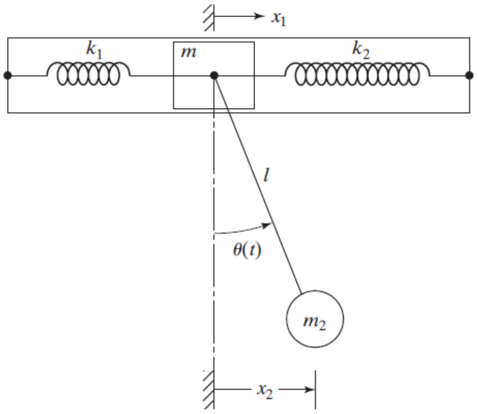
原始題目陳述如下：A two-mass system consists of a piston of mass m_1 connected by two elastic springs that moves inside a tube as shown below.　A pendulum of length l and end mass m_2 is connected to the piston as shown below. Assume that the swing angle is small so that the pendulum can be treated as linear vibration.

Please derive the equations of motion of the system in terms of $x_1(t)$ and $x_2(t)$ and find the natural frequencies of vibration of the system.

我們要問的問題:

(a) 請參考所附解答，指出該解答可能不合理或是錯誤的地方　(4 Pts)

(b) 請針對你指出錯誤的地方，進行數學修正.　(6 Pts)

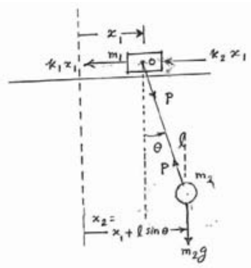


Equation of motion of the piston:

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - P \sin\theta = 0 \qquad (1)$$

Equation of motion of the pendulum bob:

$$m_2 \ddot{x}_2 + P \sin\theta = 0 \qquad (2)$$



In Eqs. (1) and (2), $\sin\theta$ can be expressed as

$$\sin\theta \simeq \theta = \frac{x_2 - x_1}{l} \qquad (3)$$

for small angles θ.

For the vertical equilibrium of mass m_2,

$$P = m_2 g \cos\theta \simeq m_2 g \qquad (4)$$

Substitution of Eqs. (3) and (4) in Eqs. (1) and (2) gives

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - m_2 g \left(\frac{x_2 - x_1}{l}\right) = 0 \qquad (5)$$

$$m_2 \ddot{x}_2 + m_2 g \left(\frac{x_2 - x_1}{l}\right) = 0 \qquad (6)$$

(c) Assuming harmonic solutions as

$$\left.\begin{aligned} x_1(t) &= X_1 \cos(\omega t + \phi) \\ x_2(t) &= X_2 \cos(\omega t + \phi) \end{aligned}\right\} \qquad (7)$$

Eqs. (5) and (6) become

$$\left\{-m_1\omega^2 + (k_1 + k_2) + \frac{m_2 g}{l}\right\} X_1 - \frac{m_2 g}{l} X_2 = 0 \qquad (8)$$

$$-\frac{m_2 g}{l} X_1 + \left\{-m_2\omega^2 + \frac{m_2 g}{l}\right\} X_2 = 0 \qquad (9)$$

By setting the determinant of the coefficient matrix of X_1 and X_2, we obtain the frequency equation as

$$\begin{vmatrix} -m_1\omega^2 & \left(k_1 + k_2 + \frac{m_2 g}{l}\right) \\ -\frac{m_2 g}{l} & \left(-m_2\omega^2 + \frac{m_2 g}{l}\right) \end{vmatrix} = 0 \qquad (10)$$

i.e.,

$$m_1 m_2 \omega^4 - \frac{m_1 m_2 g}{l}\omega^2 + \frac{m_2 g}{l}\left(k_1 + k_2 + \frac{m_2 g}{l}\right) = 0 \qquad (11)$$

The roots of Eq. (11) give the natural frequencies of vibration of the system.

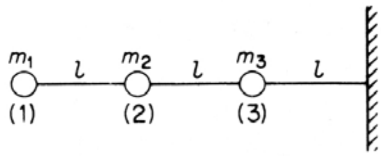
Problem 5. M-DOF Systems (II): Influence Coefficients (18 Pts)

Consider the 3DOF system constituted by three ideal beams and three concentrated masses shown below,

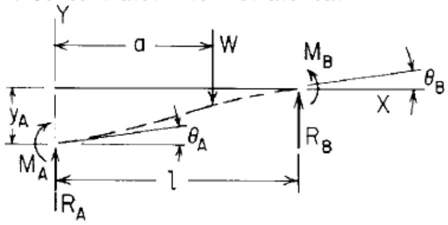
(a) **without involving any mathematical detail**; please tell us how to find their **stiffness influence** coefficients. (You may need to draw necessary figures for illustration). (4 Pts)

(b) **without involving any mathematical detail**; please tell us how to find their **flexibility influence** coefficients. (You may need to draw necessary figures for illustration). (4 Pts)

(c) Try your best to find either the stiffness or the flexibility matrices of the system if possible. (10 Pts) 下面所附的公式也許有幫助.



1. Concentrated intermediate load



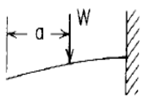
Transverse shear $= V = R_A - W\langle x-a\rangle^0$

Bending moment $= M = M_A + R_A x - W\langle x-a\rangle$

Slope $= \theta = \theta + \dfrac{M_A x}{EI} + \dfrac{R_A x^2}{2EI} - \dfrac{W}{2EI}\langle x-a\rangle^2$

Deflection $= y = y_A + \theta_A x + \dfrac{M_A x^2}{2EI} + \dfrac{R_A x^3}{6EI} - \dfrac{W}{6EI}\langle x-a\rangle^3$

(*Note*: see page 131 for a definition of the term $\langle x-a\rangle^n$.)

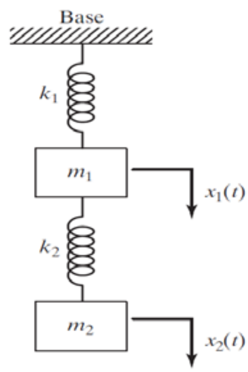
End restraints, reference no.	Boundary values	Selected maximum values of moments and deformations
1a. Left end free, right end fixed (cantilever) 	$R_A = 0 \quad M_A = 0 \quad \theta_A = \dfrac{W(l-a)^2}{2EI}$ $y_A = \dfrac{-W}{6EI}(2l^3 - 3l^2 a + a^3)$ $R_B = W \quad M_B = -W(l-a)$ $\theta_B = 0 \quad y_B = 0$	Max $M = M_B$; max possible value $= -Wl$ when $a = 0$ Max $\theta = \theta_A$; max possible value $= \dfrac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \dfrac{-Wl^3}{3EI}$ when $a = 0$

Problem 6. M-DOF Systems (III): Natural Frequencise and Modes (11 Pts)

Consider the system shown below, where m_1=m, m_2=2m, k_1=k, and k_2=2k. Gravity effect is not considered.

(a) Please find the equation of motion (5 Pts)

(b) Please find the natural frequencies and the corresponding mode shapes. (6 Pts)



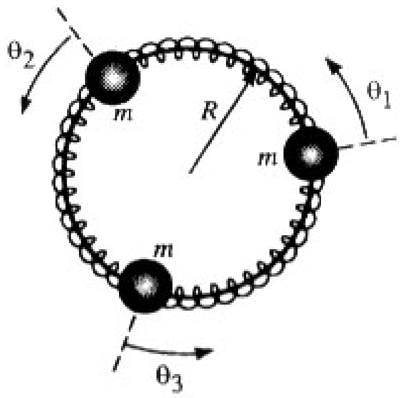
Problem 7. M-DOF Systems (IV): Natural Modes (15 Pts)

As shown in the figure below, a model of a ring molecule consists of three equal masses ***m***, which slides without friction on a fixed circular wire of radius ***R***. The masses are connected by identical springs of spring constant ***k***. The angular positions of the three masses, θ_1, θ_2, θ_3, are measured from a rest position.

(a) Use any methods, please find the equation of motion (5 Pts)

(b) 此系統有三個自然頻率, 請試著求出. (5 Pts)

(c) 此系統有三個模態. 若其中一個為 (長度 normalized 成 1) $[0.7634, -0.6325, -0.1310]^T$, 試求出另外兩個 modes (請將其長度均 normalized 成 1) (5 Pts)



Problem 8: MDOF Systems (V): Modal matrix and Principal Coordinates (10 Pts)

Consider the eigenvalue problem

$$[[k]-\omega^2[m]]\vec{X}=0\text{, where } [m]=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix},\ [k]=\begin{bmatrix}8 & -4\\ -4 & 4\end{bmatrix}.$$

Please find the natural frequencies and the principal coordinates.