

Vibration Mechanics Hw #1

(Fundamental of Vibration)

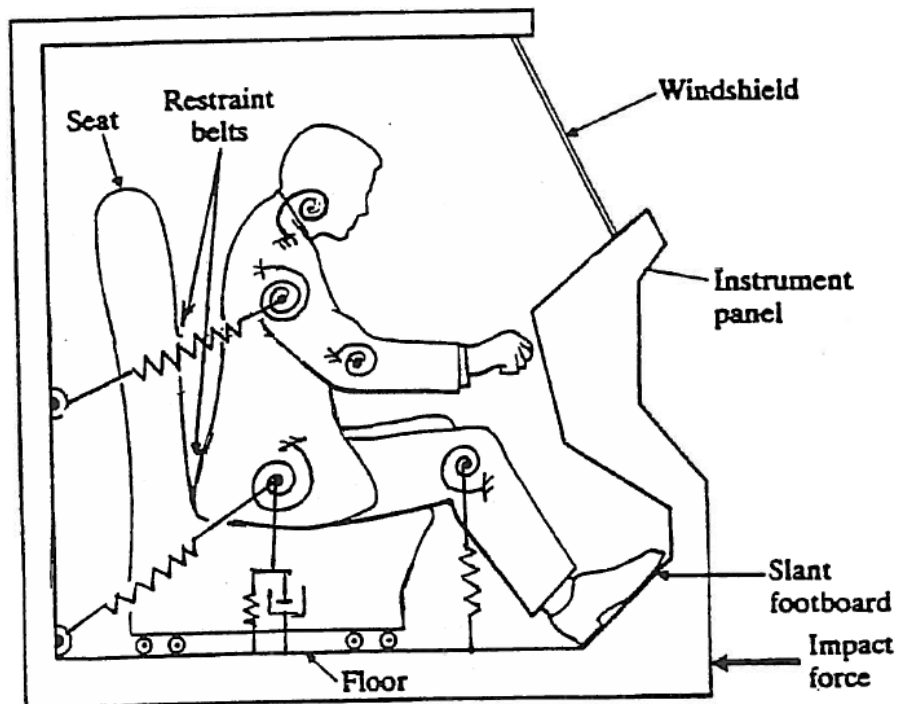
Issued: Tue. Feb. 14, 2023

Due: Fri. Mar. 03, 2023 (18:00)

1. Rao. P 1.2

Vibration problems formulation

1.2



2. Rao. P 1.9

Spring combinations

1.9 Equivalence of potential energies gives

$$\frac{1}{2} k_{t1} \theta^2 + \frac{1}{2} k_{t2} \theta^2 + \frac{1}{2} k_1 (\theta l_1)^2 + \frac{1}{2} k_2 (\theta l_1)^2 + \frac{1}{2} k_3 (\theta l_2)^2 = \frac{1}{2} k_{eq} \theta^2$$

$$\therefore k_{eq} = k_{t1} + k_{t2} + k_1 l_1^2 + k_2 l_1^2 + k_3 l_2^2$$

3. Rao. P 1.10

Spring combinations

1.10 k_{123} = for series springs k_1, k_2 and k_3 :

$$\frac{1}{k_{123}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} ; \quad k_{123} = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}$$

Using energy equivalence,

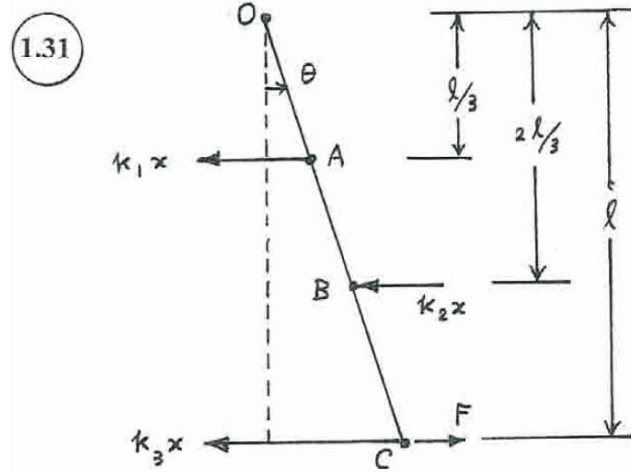
$$\frac{1}{2} k_{eq} \theta^2 = \frac{1}{2} k_4 \theta^2 + \frac{1}{2} k_{123} \theta^2 + \frac{1}{2} k_5 (\theta R)^2 + \frac{1}{2} k_6 (\theta R)^2$$

$$\therefore k_{eq} = k_4 + k_{123} + R^2 k_5 + R^2 k_6$$

$$= k_4 + \left(\frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1} \right) + R^2 (k_5 + k_6)$$

4. Rao. P. 1.31

Equivalent stiffness



Let the link OABC undergo a small angular displacement θ as shown in above figure. The spring reaction forces are also indicated in the figure. Equilibrium of moments about the pivot point O gives:

$$-k_1 x \left(\frac{l}{3} \right) - k_3 x (l) - k_2 x \left(\frac{2l}{3} \right) + F(l) = 0$$

$$\text{or } F = \left(\frac{k_1}{3} + \frac{2}{3} k_2 + k_3 \right) x \quad (1)$$

If k_{eq} denotes the equivalent spring constant of the link along the direction of F at point C, we have

$$F = k_{eq} x \quad (2)$$

Equations (1) and (2) give

$$k_{eq} = \frac{k_1}{3} + \frac{2}{3} k_2 + k_3 = \frac{k}{3} + \frac{2}{3} (2k) + (3k)$$

$$\therefore k_{eq} = \frac{14}{3} k \quad (3)$$

5. Rao. P 1.52

Equivalent mass

1.52 When point A moves by distance $x = x_h$, the walking beam rotates by the angle $\theta_b = \frac{x_h}{\ell_3}$.

This corresponds to a linear motion of point B: $x_B = \theta_b \ell_2 = \frac{x_h \ell_2}{\ell_3}$ and the angular rotation of crank can be found from the relation:

$$x_B = r_c \sin \theta_c + \ell_4 \cos \phi = r_c \sin \theta_c + \ell_4 \sqrt{1 - \frac{r_c^2}{\ell_4^2} \sin^2 \theta_c}$$

For large values of ℓ_4 compared to r_c and for small values of x and θ_c , we have

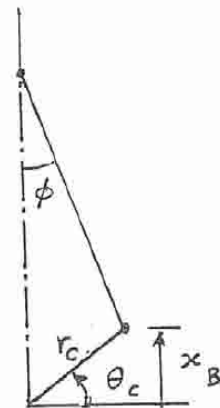
$$x_B \approx r_c \sin \theta_c = r_c \theta_c \text{ or } \theta_c = \frac{x_B}{r_c} = \frac{x_h \ell_2}{\ell_3 r_c}$$

The kinetic energy of the system can be expressed as

$$T = \frac{1}{2} m_h \dot{x}_h^2 + \frac{1}{2} J_b \dot{\theta}_b^2 + \frac{1}{2} J_c \dot{\theta}_c^2$$

Equating this to $T = \frac{1}{2} m_{eq} \dot{x}^2 = \frac{1}{2} m_{eq} \dot{x}_h^2$, we obtain

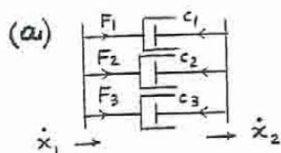
$$m_{eq} = m_h + \frac{J_b}{\ell_3^2} + J_c \left(\frac{\ell_2}{\ell_3 r_c} \right)^2$$



6. Rao. P.1.55

Equivalent damping

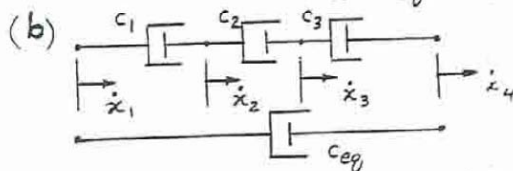
1.55)



F_i = damping force of $c_i = c_i (\dot{x}_2 - \dot{x}_1)$; $i = 1, 2, 3$

F_{eq} = damping force of $c_{eq} = c_{eq} (\dot{x}_2 - \dot{x}_1)$
 $\equiv F_1 + F_2 + F_3$

$$\therefore c_{eq} = c_1 + c_2 + c_3$$



$$F_1 = c_1 (\dot{x}_2 - \dot{x}_1)$$

$$F_2 = c_2 (\dot{x}_3 - \dot{x}_2)$$

$$F_3 = c_3 (\dot{x}_4 - \dot{x}_3)$$

$$\dot{x}_4 - \dot{x}_1 = \dot{x}_4 - \dot{x}_3 + \dot{x}_3 - \dot{x}_2 + \dot{x}_2 - \dot{x}_1$$

$$\frac{F_{eq}}{c_{eq}} = \frac{F_3}{c_3} + \frac{F_2}{c_2} + \frac{F_1}{c_1}$$

since $F_{eq} = F_1 = F_2 = F_3$, $\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$

(c) Equating the energies dissipated in a cycle,

$$\pi c_{eq} \omega X_1^2 = \pi c_1 \omega X_1^2 + \pi c_2 \omega X_2^2 + \pi c_3 \omega X_3^2$$

where $x_1 = \theta l_1$, $x_2 = \theta l_2$ and $x_3 = \theta l_3$

$$\therefore c_{eq} = c_1 + c_2 \left(\frac{l_2}{l_1}\right)^2 + c_3 \left(\frac{l_3}{l_1}\right)^2$$

(d) Equating the energies dissipated in a cycle,

$$\pi c_{teq} \omega \theta_1^2 = \pi c_{t1} \omega \theta_1^2 + \pi c_{t2} \omega \theta_2^2 + \pi c_{t3} \omega \theta_3^2$$

where $\theta_2 = \theta_1 \left(\frac{n_1}{n_2}\right)$ and $\theta_3 = \theta_1 \left(\frac{n_1}{n_3}\right)$.

$$\therefore c_{teq} = c_{t1} + c_{t2} \left(\frac{n_1}{n_2}\right)^2 + c_{t3} \left(\frac{n_1}{n_3}\right)^2$$

1.74

For two dampers in series, the equivalent damping constant is given by

$$\frac{1}{C_{eg1}} = \frac{1}{C_1} + \frac{1}{C_1} = \frac{2}{C_1}$$

$$\text{or } C_{eg1} = \frac{C_1}{2}$$

For two dampers in parallel, the equivalent damping constant is given by

$$C_{eg2} = C_2 + C_2 = 2C_2$$

Thus the system can be replaced by the two equivalent dampers in parallel as shown in the figure above.

The overall equivalent damping constant is given by

$$C_{eg} = C_{eg1} + C_{eg2} = \frac{C_1}{2} + 2C_2$$

so that

$$F = C_{eg} v$$

