Vibration Mechanics Hw #5

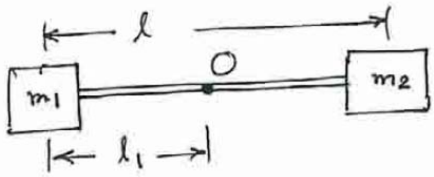
(Two-Degree-of-Freedom Systems)

Issued: May 2, 2023

Due: May 19, 2023

1. Rao P. 5.3 (20 Pts)

5.3



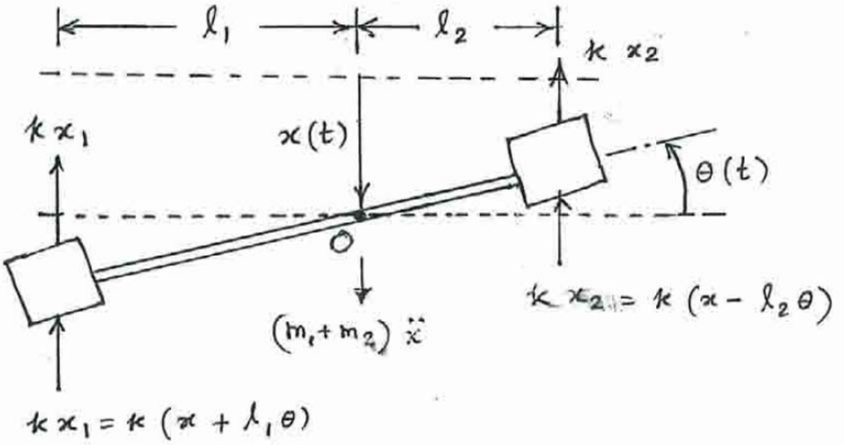
Assume masses m_1 and m_2 to be point masses.

Let O denote the C.G. of the system located at a distance of l_1 from mass m_1, so that

$$m_1 l_1 = (l - l_1)\, m_2 = l_2\, m_2$$

or $$l_1 = (l - l_1)\frac{m_2}{m_1} \quad ; \quad l_2 = l - l_1 \qquad (1)$$

Free body diagram :



Equations of motion:

$$(m_1+m_2)\,\ddot{x} = -\,2k\,(x + l_1\theta) - 2k\,(x - l_2\theta) \qquad (1)$$

$$J_o\,\ddot{\theta} = 2k\,(x - l_2\theta)\cdot l_2 - 2k\,(x + l_1\theta)\cdot l_1 \qquad (2)$$

where

(m_1+m_2) denotes the total mass acting through O

$J_o = (m_1 l_1^2 + m_2 l_2^2)$ indicates of mass moment of inertia of the system.

Eqs. (1) and (2) can be written in matrix form as

$$\begin{bmatrix} (m_1+m_2) & 0 \\ 0 & (m_1 l_1^2 + m_2 l_2^2) \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k & (2k\,l_1 - 2k\,l_2) \\ (2k\,l_1 - 2k\,l_2) & (2k\,l_2^2 + 2k\,l_1^2) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (3)$$

When $m_1 = 50$ kg, $m_2 = 200$ kg, $k = 1000$ N/m and $l = 1$ m,

Eq. (1) gives

$$l_1 = (1 - l_1)\,\frac{200}{50} = 4 - 4 l_1$$

or $l_1 = 0.8$ m ; $l_2 = 0.2$ m

Eq. (3) becomes:

$$\begin{bmatrix} 250 & 0 \\ 0 & 40 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4000 & 1200 \\ 1200 & 1360 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (4)$$

By assuming harmonic solutions

$$\left.\begin{aligned} x(t) &= X \cos(\omega t + \phi) \\ \theta(t) &= \Theta \cos(\omega t + \phi) \end{aligned}\right\} \qquad (5)$$

Eq. (4) can be written as

$$-\begin{bmatrix} 250\omega^2 & 0 \\ 0 & -40\omega^2 \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} + \begin{bmatrix} 4000 & 1200 \\ 1200 & 1360 \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (6)$$

i.e.

$$\begin{bmatrix} (4000 - 250\,\omega^2) & 1200 \\ 1200 & (1360 - 40\,\omega^2) \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (7)$$

For a nontrivial solution of X and Θ, the following condition is to be satisfied:

$$\begin{vmatrix} 4000 - 250\,\omega^2 & 1200 \\ 1200 & 1360 - 40\,\omega^2 \end{vmatrix} = 0 \tag{8}$$

Eq.(8) is the frequency equation which can be expanded as:

$$(0.01\,\omega^4 - 0.5\,\omega^2 + 4)\,10^6 = 0$$

or $$0.01\,\omega^4 - 0.5\,\omega^2 + 4 = 0 \tag{9}$$

The roots of Eq.(9) are

$$\omega^2 = \frac{+0.5 \pm \sqrt{0.25 - 0.16}}{0.02} = \frac{0.5 \pm 0.3}{0.02} = 10,\ 40$$

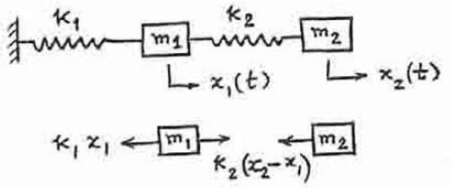
Thus the natural frequencies of vibration of the system are given by

$$\omega_1 = \sqrt{10} = 3.1622 \text{ rad/s}$$

$$\omega_2 = \sqrt{40} = 6.3245 \text{ rad/s}$$

2. Rao P. 5.21 (20 Pts)

(5.21) Equivalent system is shown in figure:



$$k_i = 2\left(\frac{24 E I_i}{h_i^3}\right) \quad ; \quad i = 1, 2$$

$$k_1 = k_2 = k = \frac{48 E I}{h^3} \; ; \; m_1 = 2m, \; m_2 = m$$

Equations of motion: $m_1 \ddot{x}_1 + (k_1 + k_2)\,x_1 - k_2 x_2 = 0$

$m_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0$

For harmonic motion $x_i(t) = X_i \cos(\omega t + \phi)$; $i = 1, 2$, we get

$$\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{---- } (E_1)$$

Frequency equation is

$$\begin{vmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{vmatrix} = 0$$

or $\omega^4 m_1 m_2 - \omega^2 (m_2 k_1 + m_2 k_2 + m_1 k_2) + k_1 k_2 = 0$

$$\omega^2 = \frac{(m_2 k_1 + m_2 k_2 + m_1 k_2) \pm \sqrt{(m_2 k_1 + m_2 k_2 + m_1 k_2)^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2} \quad \text{---} (E_2)$$

For given data,

$$\omega^2 = \frac{(mk + mk + 2mk) \pm \sqrt{(mk + mk + 2mk)^2 - 8 m^2 k^2}}{4 m^2} = \frac{k}{m}\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

$$\omega_1 = 0.5412 \sqrt{\frac{k}{m}} = 3.7495 \sqrt{\frac{EI}{m h^3}} \quad ; \quad \omega_2 = 1.3066 \sqrt{\frac{k}{m}} = 9.0524 \sqrt{\frac{EI}{m h^3}}$$

From Eq. (E_1), we get

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-\omega_1^2 m_1 + k_1 + k_2}{k_2} = \frac{-2m\,\omega_1^2 + 2k}{k} = \frac{-2(0.2929\,k) + 2k}{k}$$

$$= 1.4142$$

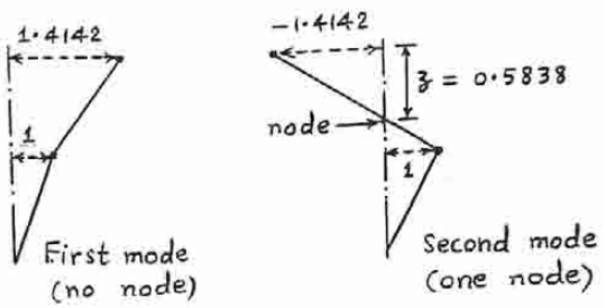
$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-\omega_2^2 m_1 + k_1 + k_2}{k_2} = \frac{-2m\,\omega_2^2 + 2k}{k} = \frac{-2(1.7071\,k) + 2k}{k}$$

$$= -1.4142$$

Mode shapes are:

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.4142 \end{Bmatrix} X_1^{(1)}$$

$$\vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ -1.4142 \end{Bmatrix} X_1^{(2)}$$

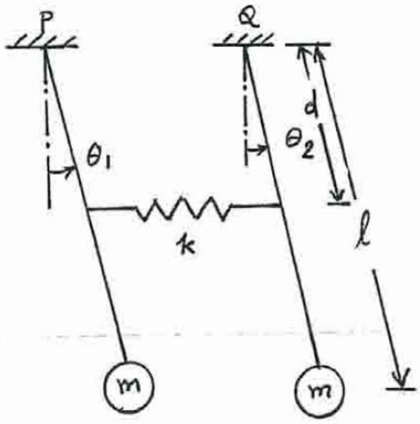


Location of node:

$$\frac{z}{1.4142} = \frac{1 - z}{1} \; ; \; z = 0.5838$$

3. Rao P. 5.31 (25 Pts)

5.31 (a) Equations of motion:



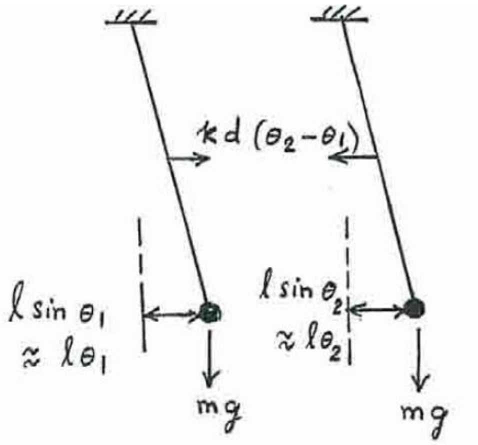
Assume: θ_1, θ_2 are small.

Moment equilibrium equations of the two masses about P and Q:

$$m\ell^2 \ddot{\theta}_1 + mg\ell\theta_1 + kd^2(\theta_1 - \theta_2) = 0 \quad (1)$$

$$m\ell^2 \ddot{\theta}_2 + mg\ell\theta_2 - kd^2(\theta_1 - \theta_2) = 0 \quad (2)$$

(b) Natural frequencies and mode shapes:



Free body diagram

Assume: Harmonic motion with

$$\theta_i(t) = \Theta_i \cos(\omega t - \phi); \; i = 1, 2 \quad (3)$$

where Θ_1 and Θ_2 are amplitudes of θ_1 and θ_2, respectively, ω is the natural frequency, and ϕ is the phase angle.

Using Eq. (3), Eqs. (1) and (2) can be expressed in matrix form as

$$-\omega^2 m\ell^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} + \begin{bmatrix} mg\ell + kd^2 & -kd^2 \\ -kd^2 & mg\ell + kd^2 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4)$$

Frequency equation:

$$\begin{vmatrix} -\omega^2 m\ell^2 + mg\ell + kd^2 & -kd^2 \\ -kd^2 & -\omega^2 m\ell^2 + mg\ell + kd^2 \end{vmatrix} = 0$$

or

$$\omega^4 - \omega^2\left(\frac{2g}{\ell} + \frac{2kd^2}{m\ell^2}\right) + \left(\frac{g^2}{\ell^2} + \frac{2gkd^2}{m\ell^3}\right) = 0 \quad (5)$$

Solution of Eq. (5) gives

$$\omega_1^2 = \frac{g}{\ell}, \quad \omega_2^2 = \frac{g}{\ell} + \frac{2kd^2}{m\ell^2} \quad (6)$$

By substituting for ω_1^2 and ω_2^2 into Eq. (4), we obtain

$$\left(\frac{\Theta_2}{\Theta_1}\right)^{(1)} = 1 \quad \text{or} \quad \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \Theta_1^{(1)}$$

and

$$\left(\frac{\Theta_2}{\Theta_1}\right)^{(2)} = -1 \quad \text{or} \quad \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \Theta_1^{(2)}$$

Thus the motion of the masses in the two modes is given by

$$\vec{\theta}^{(1)}(t) = \begin{Bmatrix} \theta_1^{(1)}(t) \\ \theta_2^{(1)}(t) \end{Bmatrix} = \Theta_1^{(1)} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \cos(\omega_1 t + \phi_1) \qquad (7)$$

$$\vec{\theta}^{(2)}(t) = \begin{Bmatrix} \theta_1^{(2)}(t) \\ \theta_2^{(2)}(t) \end{Bmatrix} = \Theta_1^{(2)} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \cos(\omega_2 t + \phi_2) \qquad (8)$$

(c) Free vibration response:

Using linear superposition of natural modes, the free vibration response of the system is given by

$$\vec{\theta}(t) = c_1\, \vec{\theta}^{(1)}(t) + c_2\, \vec{\theta}^{(2)}(t) \qquad (9)$$

By choosing $c_1 = c_2 = 1$, with no loss of generality, Eqs. (7) to (9) lead to

$$\theta_1(t) = \Theta_1^{(1)} \cos(\omega_1 t + \phi_1) + \Theta_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (10)$$

$$\theta_2(t) = \Theta_1^{(1)} \cos(\omega_1 t + \phi_1) - \Theta_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (11)$$

where $\Theta_1^{(1)}$, ϕ_1, $\Theta_1^{(2)}$ and ϕ_2 are constants to be determined from the initial conditions. When $\theta_1(0) = a$, $\theta_2(0) = 0$, $\dot{\theta}_1(0) = 0$ and $\dot{\theta}_2(0) = 0$, Eqs. (10) and (11) yield

$$\left.\begin{aligned} a &= \Theta_1^{(1)} \cos\phi_1 + \Theta_1^{(2)} \cos\phi_2 \\ 0 &= \Theta_1^{(1)} \cos\phi_1 - \Theta_1^{(2)} \cos\phi_2 \\ 0 &= -\omega_1 \Theta_1^{(1)} \sin\phi_1 - \omega_2 \Theta_1^{(2)} \sin\phi_2 \\ 0 &= -\omega_1 \Theta_1^{(1)} \sin\phi_1 + \omega_2 \Theta_1^{(2)} \sin\phi_2 \end{aligned}\right\} \qquad (12)$$

Eqs. (12) can be solved for $\Theta_1^{(1)}$, ϕ_1, $\Theta_1^{(2)}$ and ϕ_2 to obtain

$$\left.\begin{aligned} \theta_1(t) &= a \cos\frac{\omega_2 - \omega_1}{2} t \cdot \cos\frac{\omega_2 + \omega_1}{2} t \\ \theta_2(t) &= a \sin\frac{\omega_2 - \omega_1}{2} t \cdot \sin\frac{\omega_2 + \omega_1}{2} t \end{aligned}\right\} \qquad (13)$$

(d) Conditions for beating:

When $\frac{2k\,d^2}{m\ell^2} \ll \frac{g}{\ell}$ or $k \ll \frac{mg\ell}{2d^2}$, (14)

the two frequency components in Eqs. (13), namely, $\frac{\omega_2-\omega_1}{2}$ and $\frac{\omega_2+\omega_1}{2}$, can be approximated as

$$\Omega_1 = \frac{\omega_2-\omega_1}{2} \simeq \frac{k}{2m}\frac{d^2}{\sqrt{g\ell^3}} \tag{15}$$

and

$$\Omega_2 = \frac{\omega_2+\omega_1}{2} \simeq \sqrt{\frac{g}{\ell}} + \frac{k}{2m}\frac{d^2}{\sqrt{g\ell^3}} \tag{16}$$

This implies that the motions of the pendulums are given by

$$\left.\begin{aligned} \theta_1(t) &\simeq a\cos\Omega_1 t\cdot\cos\Omega_2 t \\ \theta_2(t) &\simeq a\sin\Omega_1 t\cdot\sin\Omega_2 t \end{aligned}\right\} \tag{17}$$

This motion, Eqs. (17), denotes beating phenomenon.

4. Rao P. 5.37 (10 Pts)

5.37 Equation of motion of mass m: $m\ddot{x} = -k_2(x - r\theta)$ ---(E_1)

Equation of motion of cylinder of mass m_0 and mass moment of inertia $J_0 = \frac{1}{2} m_0 r^2$: $J_0 \ddot{\theta} = -k_1 r^2 \theta - k_2 (r\theta - x) r$ ---(E_2)

For $x(t) = X \cos(\omega t + \phi)$ and $\theta(t) = \Theta \cos(\omega t + \phi)$, Eqs. (E_1) and (E_2) give the frequency equation

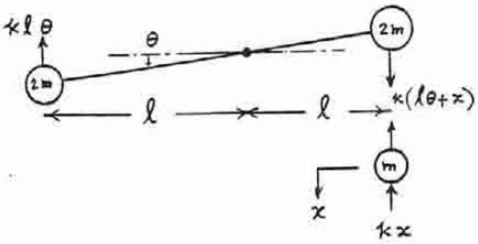
$$\begin{vmatrix} -m\omega^2 + k_2 & -k_2 r \\ -k_2 r & -\frac{1}{2} m_0 r^2 \omega^2 + k_1 r^2 + k_2 r^2 \end{vmatrix} = 0$$

i.e. $$\omega^4 - \omega^2\left(\frac{k_2}{m} + \frac{2\{k_1 + k_2\}}{m_0}\right) + \frac{2 k_1 k_2}{m_0 m} = 0$$

$$\omega_1^2, \omega_2^2 = \frac{k_2}{2m} + \frac{(k_1 + k_2)}{m_0} \mp \sqrt{\frac{1}{4}\left(\frac{k_2}{m} + \frac{2k_1}{m_0} + \frac{2k_2}{m_0}\right)^2 - \frac{2 k_1 k_2}{m\, m_0}}$$

5. Rao P. 5.42 (10 Pts)

5.42 Equations of motion:



$$4m\ell^2\ddot{\theta} = -k\ell\theta \cdot \ell - k(\ell\theta + x)\ell$$

$$m\ddot{x} = -kx - k(\ell\theta + x)$$

i.e. $$4m\ell^2\ddot{\theta} + 2k\ell^2\theta + k\ell x = 0$$

$$m\ddot{x} + 2kx + k\ell\theta = 0$$

For harmonic motion, these equations give

$$\begin{bmatrix} -4m\ell^2\omega^2 + 2k\ell^2 & k\ell \\ k\ell & -m\omega^2 + 2k \end{bmatrix} \begin{Bmatrix} \Theta \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is $4m^2\omega^4 - 10\,km\,\omega^2 + 3k^2 = 0$

$$\omega^2 = \frac{k}{m}\left(\frac{5}{4} \mp \frac{\sqrt{13}}{4}\right) = 0.3486\frac{k}{m}, \quad 2.1514\frac{k}{m}$$

$$\omega_1 = 0.5904\sqrt{\frac{k}{m}}, \quad \omega_2 = 1.4668\sqrt{\frac{k}{m}}$$

Amplitude ratios are

$$r_1 = \frac{X^{(1)}}{\Theta^{(1)}} = \frac{-4m\ell^2\omega_1^2 + 2k\ell^2}{-k\ell} = -0.6056\,\ell$$

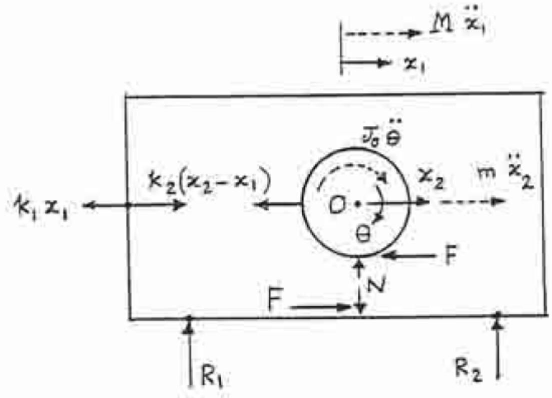
$$r_2 = \frac{X^{(2)}}{\Theta^{(2)}} = \frac{-4m\ell^2\omega_2^2 + 2k\ell^2}{-k\ell} = 6.6056\,\ell$$

Mode shapes are $$\vec{X}^{(1)} = \begin{Bmatrix} \Theta^{(1)} \\ X^{(1)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.6056\ell \end{Bmatrix} \Theta^{(1)}$$

$$\vec{X}^{(2)} = \begin{Bmatrix} \Theta^{(2)} \\ X^{(2)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 6.6056\,\ell \end{Bmatrix} \Theta^{(2)}$$

6. Rao P. 5.51 (10 Pts)

5.51



Free body diagram

N = normal reaction between cylinder and trailer, F = friction force, R_1, R_2 = reactions between trailer and ground.

Equation of motion for linear motion of cylinder:

$$\sum F = m\,\ddot{x}_2 \quad \text{or} \quad m\,\ddot{x}_2 = -F - k_2\,(x_2 - x_1) \tag{1}$$

Equation of motion for rotational motion of cylinder:

$$\sum M_0 = J_0\,\ddot{\theta} \quad \text{or} \quad J_0\,\ddot{\theta} = F\,r \tag{2}$$

where $J_0 = \frac{1}{2}\,m\,r^2$ and $\theta = \frac{x_2 - x_1}{r}$.

Equation of motion for linear motion of trailer:

$$\sum F = M\,\ddot{x}_1 \quad \text{or} \quad M\,\ddot{x}_1 = -k_1\,x_1 + k_2\,(x_2 - x_1) + F \tag{3}$$

Eq. (2) gives

$$F = \frac{J_0\,\ddot{\theta}}{r} = \frac{1}{r}\,(\frac{1}{2}\,m\,r^2)\left(\frac{\ddot{x}_2 - \ddot{x}_1}{r}\right) = \frac{m}{2}\,(\ddot{x}_2 - \ddot{x}_1) \tag{4}$$

Substitution of Eq. (4) into Eqs. (1) and (3) yields the equations of motion as:

$$\frac{3\,m}{2}\,\ddot{x}_2 - \frac{1}{2}\,m\,\ddot{x}_1 - k_2\,x_1 + k_2\,x_2 = 0 \tag{5}$$

$$(M + \frac{m}{2})\,\ddot{x}_1 - \frac{m}{2}\,\ddot{x}_2 + x_1\,(k_1 + k_2) - k_2\,x_2 = 0 \tag{6}$$

7. Rao P. 5.58 (15 Pts)

5.58 Equations of motion:

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = F_1(t) = F_0 \sin \omega t \quad \text{----} \; (E_1)$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \quad \text{----} \; (E_2)$$

We use $F_0 e^{i\omega t}$ (with $i = \sqrt{-1}$) for $F_1(t)$ and consider only the imaginary part at the end.

Let $x_j(t) = X_j e^{i\omega t}$; $j = 1, 2$

Eqs. (E_1) and (E_2) become

$$-m_1 \omega^2 X_1 e^{i\omega t} + (k_1 + k_2) X_1 e^{i\omega t} - k_2 X_2 e^{i\omega t} = F_0 e^{i\omega t}$$

$$-m_2 \omega^2 X_2 e^{i\omega t} + k_2 X_2 e^{i\omega t} - k_2 X_1 e^{i\omega t} = 0$$

i.e. $$[Z(i\omega)] \vec{X} = \underset{\sim}{\vec{F}_0} \quad \text{----} \; (E_3)$$

where $\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$, $\underset{\sim}{\vec{F}_0} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$,

$$[Z(i\omega)] = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{21}(i\omega) & Z_{22}(i\omega) \end{bmatrix},$$

$Z_{11}(i\omega) = -m_1 \omega^2 + k_1 + k_2$, $Z_{12}(i\omega) = Z_{21}(i\omega) = -k_2$,

$Z_{22}(i\omega) = -m_2 \omega^2 + k_2$.

Eqs. (5.35) give

$$X_1(i\omega) = \frac{(-m_2 \omega^2 + k_2) F_0}{(-m_1 \omega^2 + k_1 + k_2)(-m_2 \omega^2 + k_2) - k_2^2}$$

$$X_2(i\omega) = \frac{k_2 F_0}{(-m_1 \omega^2 + k_1 + k_2)(-m_2 \omega^2 + k_2) - k_2^2}$$

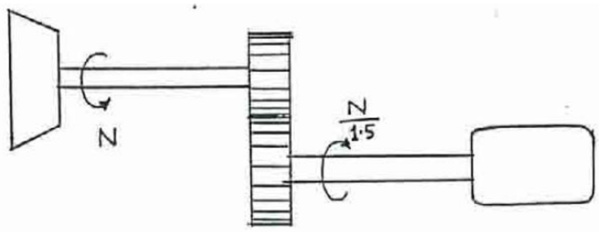
Since $F_0 \sin \omega t = \mathrm{Im}\,(F_0 e^{i\omega t})$, $x_j(t) = \mathrm{Im}\,(X_j e^{i\omega t}) = X_j \sin \omega t$

$$\therefore \; x_1(t) = \frac{(-m_2 \omega^2 + k_2) F_0}{(-m_1 \omega^2 + k_1 + k_2)(-m_2 \omega^2 + k_2) - k_2^2} \sin \omega t$$

$$x_2(t) = \frac{k_2 F_0}{(-m_1 \omega^2 + k_1 + k_2)(-m_2 \omega^2 + k_2) - k_2^2} \sin \omega t$$

8. Rao P. 5.70 (10 Pts)

5.70



Since the length of shaft 1 is small and its diameter large, it will be very rigid and hence the turbine and gear 1 are assumed to be rigidly connected. This helps in modeling the system as a two d.o.f. system.

$$J_{01} = J_{turbine} + J_{gear1} + \frac{J_{gear2}}{1.5^2} = 3000 + 500 + (1000/2.25) = 3944.4444 \text{ kg}-\text{m}^2$$

$$k_{t2} = \left(\frac{GJ}{\ell}\right)_{shaft2} = \frac{(80\,(10^9))\left(\frac{\pi}{32}\,(0.1^4)\right)}{1} = 7.854\,(10^5) \text{ N/m}$$

$$J_{02} = J_{generator} = 2000 \text{ kg}-\text{m}^2$$

System is a semi-definite system. Its natural frequencies are given by (see Eq. (5.40)):

$$\omega_1 = 0$$

$$\omega_2 = \sqrt{\frac{k_{t2}\,(J_{01} + J_{02})}{J_{01}\,J_{02}}} = \sqrt{\frac{(78.54\,(10^4))\,(5944.4444)}{(3944.4444)\,(2000)}} = 24.3273 \text{ rad/sec}$$