# Vibration Mechanics Hw #6

## (Multiple-Degree-of-Freedom Systems)

**145 Pts**

Issued: May 19 (FRI), 2023

Due: June 05 (MON), 2023

**1.** Rao P. 6.2 (please refer to Figure 6.19) 15 Pts

Equations of motion:

$$J_0\,\ddot{\theta} = -2\,k\left(\frac{\ell}{4}\,\theta - x_1\right)\frac{\ell}{4} - c\left(\frac{\ell}{4}\,\dot{\theta} - \dot{x}_1\right)\frac{\ell}{4} - 3\,k\,(\theta\,\ell)\,\ell + M_t$$

$$2\,m\,\ddot{x}_1 = -2\,k\left(x_1 - \frac{\ell}{4}\,\theta\right) - c\left(\dot{x}_1 - \frac{\ell}{4}\,\dot{\theta}\right) - k\,(x_1 - x_2) + F_1$$

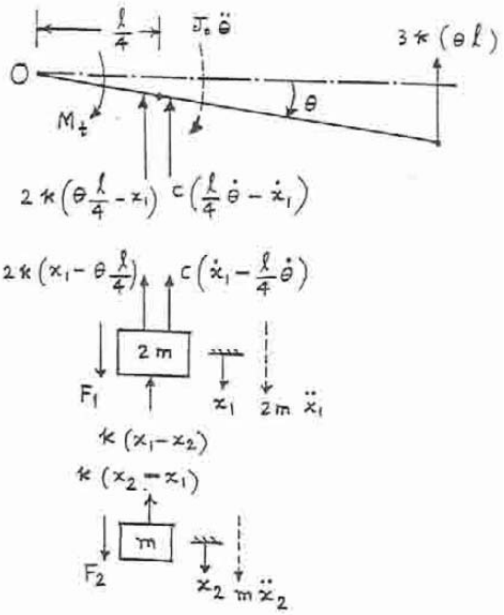
$$m\,\ddot{x}_2 = -k\,(x_2 - x_1) + F_2$$

$$\text{where}\quad J_0 = \frac{1}{3}\,(2\,m)\,\ell^2 = \frac{2}{3}\,m\,\ell^2$$

These equations can be stated in matrix form as:

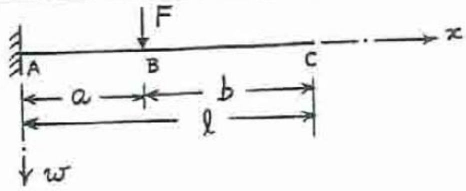
$$\begin{bmatrix} \frac{2}{3}\,m\,\ell^2 & 0 & 0 \\ 0 & 2\,m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \frac{c\,\ell^2}{16} & -\frac{c\,\ell}{4} & 0 \\ -\frac{c\,\ell}{4} & c & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+\begin{bmatrix} \frac{25\,k\,\ell^2}{8} & -\frac{k\,\ell}{2} & 0 \\ -\frac{k\,\ell}{2} & 3\,k & -k \\ 0 & -k & k \end{bmatrix}\begin{Bmatrix} \theta \\ x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} M_t \\ F_1 \\ F_2 \end{Bmatrix}$$



**2.** Rao P. 6.25 15 Pts

From strength of materials, the deflection of the cantilever beam shown is given by




$$w(x)\Big|_{\text{in } AB} = \frac{Fx^2}{6EI}(-x+3a) \quad --- (E_1)$$

$$w(x)\Big|_{\text{in } BC} = \frac{Fa^2}{6EI}(-a+3x) \quad --- (E_2)$$

Apply $F_1 = 1$, $F_2 = F_3 = 0$ : $a_{11} = (F=1,\ x=l,\ a=l \text{ in } (E_1)) = l^3/(3EI)$

$a_{21} = (F=1,\ x=2l,\ a=l \text{ in } (E_2)) = 5l^3/(6EI)$

$a_{31} = (F=1,\ x=3l,\ a=l \text{ in } (E_2)) = 4l^3/(3EI)$

Similarly apply $F_2 = 1$, $F_1 = F_3 = 0$ to get $a_{22}, a_{32}, a_{12}$ and $F_3 = 1$, $F_1 = F_2 = 0$ to get $a_{33}, a_{13}, a_{23}$. Result is

$$[a] = \frac{l^3}{EI}\begin{bmatrix} 1/3 & 5/6 & 4/3 \\ 5/6 & 8/3 & 14/3 \\ 4/3 & 14/3 & 9 \end{bmatrix}$$

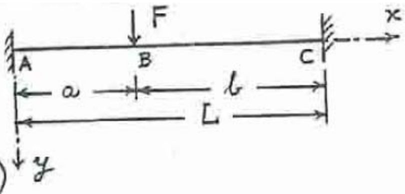
Equations of motion:

$$[m]\ \ddot{\vec{w}} + [k]\ \vec{w} = \vec{0} \quad \text{or} \quad [a][m]\ \ddot{\vec{w}} + \vec{w} = \vec{0}$$

with $[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$ and $\vec{w} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}$.

**3.** Rao P. 6.26 15 Pts

Deflection of a fixed-fixed beam is




$$y(x)\Big|_{\text{in } AB} = \frac{Fb^2x^2}{6EIL^3}\left\{-x(3a+b)+3aL\right\} \text{ --- } (E_1)$$

$$y(x)\Big|_{\text{in } BC} = \frac{Fa^2(L-x)^2}{6EIL^3}\left\{-(L-x)(3b+a)+3bL\right\} \text{ --- } (E_2)$$

Apply $F_1 = 1$, $F_2 = F_3 = 0$:

$a_{11} = (F=1,\ a=l,\ b=3l,\ x=l,\ L=4l$ in $(E_1)) = 9l^3/(64EI)$

$a_{21} = (F=1,\ a=l,\ b=3l,\ x=2l,\ L=4l$ in $(E_2)) = l^3/(6EI)$

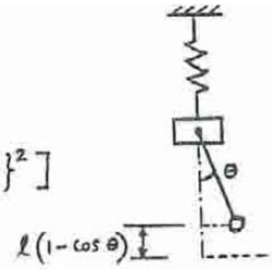
$a_{31} = (F=1,\ a=l,\ b=3l,\ x=3l,\ L=4l$ in $(E_2)) = 13l^3/(192EI)$

Similarly apply $F_2=1$, $F_1=F_3=0$ to get $a_{22}, a_{32}, a_{12}$ and $F_3=1$, $F_1=F_2=0$ to get $a_{33}, a_{13}, a_{23}$. Result is

$$[a] = \frac{l^3}{EI}\begin{bmatrix} 9/64 & 1/6 & 13/192 \\ 1/6 & 1/3 & 1/6 \\ 13/192 & 1/6 & 9/64 \end{bmatrix}$$

**4.** Rao P. 6.39 20 Pts

Coordinates of the bob are $(x + l\cos\theta,\ l\sin\theta)$




T = kinetic energy = kinetic energy of slider + kinetic energy of bob

$$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}m\left[\left\{\frac{d}{dt}(x + l\cos\theta)\right\}^2 + \left\{\frac{d}{dt}(l\sin\theta)\right\}^2\right]$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2(\sin^2\theta + \cos^2\theta) - \frac{1}{2}m(2\dot{x}\,l\sin\theta\ \dot{\theta})$$

$$= m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - m\dot{x}\dot{\theta}\,l\sin\theta \simeq m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 \text{ for small } \theta.$$

V = potential energy = potential energy of spring + potential energy of bob

$$= \frac{1}{2}kx^2 + mgl(1-\cos\theta)$$

(Note: Potential energy of slider need not be considered if $x=0$ corresponds to static equilibrium position)

Since $\cos\theta \simeq 1 - \frac{1}{2}\theta^2$, $V = \frac{1}{2}kx^2 + \frac{1}{2}mgl\theta^2$

As there are no external forces, Lagrange's equations become

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0\ ; \quad j = 1, 2$$

Here $q_1 = x$ and $q_2 = \theta$

$$\frac{\partial T}{\partial x} = 0,\quad \frac{\partial T}{\partial \dot{x}} = 2m\dot{x},\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = 2m\ddot{x},\quad \frac{\partial V}{\partial x} = kx$$

$$\frac{\partial T}{\partial \theta} = 0,\quad \frac{\partial T}{\partial \dot{\theta}} = ml^2\dot{\theta},\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta},\quad \frac{\partial V}{\partial \theta} = mgl\theta$$

Lagrange's equations become

$$2m\ddot{x} + kx = 0\ ; \quad ml^2\ddot{\theta} + mgl\theta = 0 \text{ or } l\ddot{\theta} + g\theta = 0$$

**5.** P. 6.40 20 Pts

(1) With $x_1$ and $x_2$ as generalized coordinates:

Since $x_1 = x - l_1\theta$ and $x_2 = x + l_2\theta$,

$$x = \left(\frac{x_1 l_2 + x_2 l_1}{l_1 + l_2}\right) \quad \text{and} \quad \theta = \left(\frac{x_2 - x_1}{l_1 + l_2}\right)$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 = \frac{1}{2} m \left(\frac{\dot{x}_1 l_2 + \dot{x}_2 l_1}{l_1 + l_2}\right)^2 + \frac{1}{2} J_0 \left(\frac{\dot{x}_2 - \dot{x}_1}{l_1 + l_2}\right)^2$$

$$\frac{\partial T}{\partial x_1} = 0, \quad \frac{\partial T}{\partial \dot{x}_1} = \frac{m}{2(l_1+l_2)^2}\left(2 l_2^2 \dot{x}_1 + 2 l_1 l_2 \dot{x}_2\right) + \frac{J_0}{2(l_1+l_2)^2}\left(2\dot{x}_1 - 2\dot{x}_2\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = \frac{m}{(l_1+l_2)^2}\left(l_2^2 \ddot{x}_1 + l_1 l_2 \ddot{x}_2\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_1 - \ddot{x}_2\right)$$

$$\frac{\partial T}{\partial x_2} = 0, \quad \frac{\partial T}{\partial \dot{x}_2} = \frac{m}{2(l_1+l_2)^2}\left(2 l_1^2 \dot{x}_2 + 2 l_1 l_2 \dot{x}_1\right) + \frac{J_0}{2(l_1+l_2)^2}\left(2\dot{x}_2 - 2\dot{x}_1\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = \frac{m}{(l_1+l_2)^2}\left(l_1^2 \ddot{x}_2 + l_1 l_2 \ddot{x}_1\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_2 - \ddot{x}_1\right)$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k\, x_2^2$$

$$\frac{\partial V}{\partial x_1} = k_1 x_1, \quad \frac{\partial V}{\partial x_2} = k_2 x_2$$

Lagrange's equations, Eq. (6.41), give

$$\frac{m}{(l_1+l_2)^2}\left(l_2^2 \ddot{x}_1 + l_1 l_2 \ddot{x}_2\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_1 - \ddot{x}_2\right) + k_1 x_1 = 0$$

$$\frac{m}{(l_1+l_2)^2}\left(l_1^2 \ddot{x}_2 + l_1 l_2 \ddot{x}_1\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_2 - \ddot{x}_1\right) + k_2 x_2 = 0$$

i.e.

$$\ddot{x}_1\left(m l_2^2 + J_0\right) + \ddot{x}_2\left(m l_1 l_2 - J_0\right) + x_1 (l_1 + l_2)^2 k_1 = 0$$

$$\ddot{x}_1\left(m l_1 l_2 - J_0\right) + \ddot{x}_2\left(m l_1^2 + J_0\right) + x_2 (l_1 + l_2)^2 k_2 = 0$$

(2) With $x$ and $\theta$ as generalized coordinates:

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2$$

$$V = \frac{1}{2} k_1 (x - l_1\theta)^2 + \frac{1}{2} k_2 (x + l_2\theta)^2$$

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial \dot{x}} = m\dot{x}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\ddot{x}, \quad \frac{\partial V}{\partial x} = k_1(x - l_1\theta) + k_2(x + l_2\theta)$$

$$\frac{\partial T}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \dot{\theta}} = J_0\dot{\theta}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = J_0\ddot{\theta}, \quad \frac{\partial V}{\partial \theta} = -k_1 l_1 (x - l_1\theta) + k_2 l_2 (x + l_2\theta)$$

Lagrange's equations give

$$m\ddot{x} + k_1(x - l_1\theta) + k_2(x + l_2\theta) = 0$$

$$J_0\ddot{\theta} - k_1 l_1 (x - l_1\theta) + k_2 l_2 (x + l_2\theta) = 0$$

**6.** Rao P. 6.75 15Pts

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 3m \end{bmatrix}, \quad [k] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1+k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

Equations of motion for harmonic motion:

$$\begin{bmatrix} (-m\omega^2+k) & -k & 0 \\ -k & (-2m\omega^2+2k) & -k \\ 0 & -k & (-3m\omega^2+k) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E.1)$$

Defining $\alpha = \frac{m\omega^2}{k}$, (E.1) can be rewritten as

$$\begin{bmatrix} (-\alpha+1) & -1 & 0 \\ -1 & (-2\alpha+2) & -1 \\ 0 & -1 & (-3\alpha+1) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E.2)$$

Frequency equation is

$$2\alpha\,(3\alpha^2 - 7\alpha + 3) = 0$$

Roots are

$\alpha_1 = 0$ ; $\omega_1 = 0$

$\alpha_2 = 0.565741$ ; $\omega_2 = 0.752158\sqrt{k/m}$

$\alpha_3 = 1.767592$ ; $\omega_3 = 1.329508\sqrt{k/m}$

Eqs. (E.2) give $\quad X_2^{(j)} = (-\alpha_j+1)\,X_1^{(j)}, \quad X_3^{(j)} = \left(\frac{1}{-3\alpha_j+1}\right) X_2^{(j)}$

$$\therefore \vec{X}^{(j)} = \begin{Bmatrix} 1.0 \\ (-\alpha_j+1) \\ \left(\frac{-\alpha_j+1}{-3\alpha_j+1}\right) \end{Bmatrix} X_1^{(j)}$$

Hence

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} X_1^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0.434259 \\ -0.622841 \end{Bmatrix} X_1^{(2)}, \quad \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -0.767592 \\ 0.178395 \end{Bmatrix} X_1^{(3)}$$

**7.** Rao P. 6.77 20 Pts

(6.77) From solution of Problem 6.57, the natural frequencies and mode shapes are given by:

$$\omega_1 = 0.482087\sqrt{\frac{k}{m}} \;;\; \omega_2 = \sqrt{\frac{k}{m}} \;;\; \omega_3 = 1.197605\sqrt{\frac{k}{m}}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.535184 \\ 1.0 \end{Bmatrix} \;;\; \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \;;\; \vec{X}^{(3)} = \begin{Bmatrix} 1.0 \\ 0.868516 \\ -1 \end{Bmatrix}$$

Initial conditions:

$$x_1(0) = x_{10},\; x_2(0) = 0,\; x_3(0) = 0,\; \dot{x}_i(0) = 0 \;;\; i = 1, 2, 3$$

Equations (6.98) and (6.99) yield:

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = x_{10} \quad (1)$$

$$1.5352\, A_1 \cos\phi_1 + 0.8685\, A_3 \cos\phi_3 = 0 \quad (2)$$

$$A_1 \cos\phi_1 - A_2 \cos\phi_2 - A_3 \cos\phi_3 = 0 \quad (3)$$

$$-0.4821\sqrt{\frac{k}{m}}\, A_1 \sin\phi_1 - \sqrt{\frac{k}{m}}\, A_2 \sin\phi_2 - 1.1976\sqrt{\frac{k}{m}}\, A_3 \sin\phi_3 = 0 \quad (4)$$

$$-0.4821\sqrt{\frac{k}{m}}\,(1.5352)\, A_1 \sin\phi_1 - 1.1976\sqrt{\frac{k}{m}}\,(0.8685)\, A_3 \sin\phi_3 = 0 \quad (5)$$

$$-0.4821\sqrt{\frac{k}{m}}\,(1.0)\, A_1 \sin\phi_1 - \sqrt{\frac{k}{m}}\,(-1)\, A_2 \sin\phi_2 - 1.1976\sqrt{\frac{k}{m}}\,(-1)\, A_3 \sin\phi_3 = 0 \quad (6)$$

Solution of Eqs. (4) to (6):

$$\phi_i = 0 \;;\; i = 1, 2, 3$$

Solution of Eqs. (1) to (3) gives:

$$A_1 = 0.5\, x_{10} \;;\; A_2 = 0.3838\, x_{10} \;;\; A_3 = -0.8838\, x_{10}$$

Thus the free vibration solution of the system is given by:

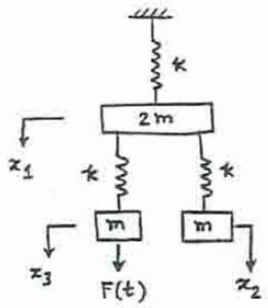
$$x_1(t) = x_{10}\left(0.5 \cos 0.4821\sqrt{\frac{k}{m}}\, t + 0.3838 \cos\sqrt{\frac{k}{m}}\, t - 0.8838 \cos 1.1976\sqrt{\frac{k}{m}}\, t\right)$$

$$x_2(t) = x_{10}\left\{0.7676 \cos 0.4821\sqrt{\frac{k}{m}}\, t - 0.7676 \cos 1.1976\sqrt{\frac{k}{m}}\, t\right\}$$

$$x_3(t) = x_{10}\left(0.5 \cos 0.4821\sqrt{\frac{k}{m}}\, t - 0.3838 \cos\sqrt{\frac{k}{m}}\, t + 0.8838 \cos 1.1976\sqrt{\frac{k}{m}}\, t\right)$$

**8.** Rao P. 6.89 25 Pts

(6.89) Equations of motion are




$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 3k & -k & -k \\ -k & k & 0 \\ -k & 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F(t) \end{Bmatrix}$$

where $m = 1$ kg, $k = 1000$ N/m, $F(t) = 5 \sin 10t$ N.

Eigenvalue analysis:

Frequency equation is

$$\left| -\omega^2 m \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + k \begin{bmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \right| = 0$$

or $$\begin{vmatrix} -2\lambda+3 & -1 & -1 \\ -1 & -\lambda+1 & 0 \\ -1 & 0 & -\lambda+1 \end{vmatrix} = 0 \quad \text{where } \lambda = \frac{\omega^2 m}{k}$$

or $$2\lambda^3 - 7\lambda^2 + 6\lambda - 1 = 0$$

Roots are

$\lambda_1 = 0.219220$, $\lambda_2 = 1.0$, $\lambda_3 = 2.28078$

$\omega_1 = 0.4682094 \sqrt{\frac{k}{m}}$, $\omega_2 = 1.0 \sqrt{\frac{k}{m}}$, $\omega_3 = 1.5102251 \sqrt{\frac{k}{m}}$

Mode shapes are given by

$$\begin{bmatrix} -2\lambda_i+3 & -1 & -1 \\ -1 & -\lambda_i+1 & 0 \\ -1 & 0 & -\lambda_i+1 \end{bmatrix} \begin{Bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

or $X_1^{(i)} = (-\lambda_i+1)\, X_2^{(i)}$, $X_3^{(i)} = (-2\lambda_i+3)\, X_1^{(i)} - X_2^{(i)}$

$$\vec{X}^{(i)} = \begin{Bmatrix} -\lambda_i+1 \\ 1 \\ (-2\lambda_i+3)(-\lambda_i+1) - 1 \end{Bmatrix} X_2^{(i)}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 0.78078 \\ 1 \\ 1 \end{Bmatrix} X_2^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} X_2^{(2)}, \quad \vec{X}^{(3)} = \begin{Bmatrix} -1.28078 \\ 1 \\ 1 \end{Bmatrix} X_2^{(3)}$$

Normalization of mode shapes with respect to [m]:

$$\vec{X}^{(1)T} [m] \vec{X}^{(1)} = \begin{pmatrix} 0.78078 & 1 & 1 \end{pmatrix} \begin{Bmatrix} 1.56156 \\ 1 \\ 1 \end{Bmatrix} \left(X_2^{(1)}\right)^2 = 3.21923 \left(X_2^{(1)}\right)^2 = 1$$

$$X_2^{(1)} = 0.55734 \; ; \quad \vec{X}^{(1)} = \begin{Bmatrix} 0.43516 \\ 0.55734 \\ 0.55734 \end{Bmatrix}$$

$$\vec{x}^{(2)T}[m]\,\vec{x}^{(2)} = \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} \left(x_2^{(2)}\right)^2 = 2\left(x_2^{(2)}\right)^2 = 1$$

$$x_2^{(2)} = 0.70711 \; ; \quad \vec{x}^{(2)} = \begin{Bmatrix} 0 \\ 0.70711 \\ -0.70711 \end{Bmatrix}$$

$$\vec{x}^{(3)T}[m]\,\vec{x}^{(3)} = \begin{pmatrix} -1.28078 & 1 & 1 \end{pmatrix} \begin{Bmatrix} -2.56156 \\ 1 \\ 1 \end{Bmatrix} \left(x_2^{(3)}\right)^2 = 5.28079\left(x_2^{(3)}\right)^2 = 1$$

$$x_2^{(3)} = 0.43516 \; ; \quad \vec{x}^{(3)} = \begin{Bmatrix} -0.55734 \\ 0.43516 \\ 0.43516 \end{Bmatrix}$$

$\vec{Q} = [X]^T \vec{F}(t) =$ vector of generalized forces

$$= \begin{bmatrix} 0.43516 & 0.55734 & 0.55734 \\ 0 & 0.70711 & -0.70711 \\ -0.55734 & 0.43516 & 0.43516 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ F_0 \sin \omega t \end{Bmatrix} = \begin{Bmatrix} 2.7867 \\ -3.5356 \\ 2.1758 \end{Bmatrix} \sin 10t$$

Uncoupled equations of motion are

$$\left.\begin{aligned} \ddot{q}_1 + 219.22\, q_1 &= 2.7867 \sin 10t \\ \ddot{q}_2 + 1000.00\, q_2 &= -3.5356 \sin 10t \\ \ddot{q}_3 + 2280.78\, q_3 &= 2.1758 \sin 10t \end{aligned}\right\} \qquad (E.1)$$

Particular solutions of (E.1) are

$$q_1(t) = \left(\frac{2.7867}{219.22 - 100}\right) \sin 10t = 0.0233744 \sin 10t$$

$$q_2(t) = \left(\frac{-3.5356}{1000 - 100}\right) \sin 10t = -0.0039284 \sin 10t$$

$$q_3(t) = \left(\frac{2.1758}{2280.78 - 100}\right) \sin 10t = 0.0009977 \sin 10t$$

Since $\vec{x} = [X]\,\vec{q} = \begin{bmatrix} 0.43516 & 0 & -0.55734 \\ 0.55734 & 0.70711 & 0.43516 \\ 0.55734 & -0.70711 & 0.43516 \end{bmatrix} \vec{q}$, we get

$$x_1(t) = 0.0096155 \sin 10t \quad m$$

$$x_2(t) = 0.0095333 \sin 10t \quad m$$

$$x_3(t) = 0.0162395 \sin 10t \quad m.$$