Vibration Mechanics Hw #1

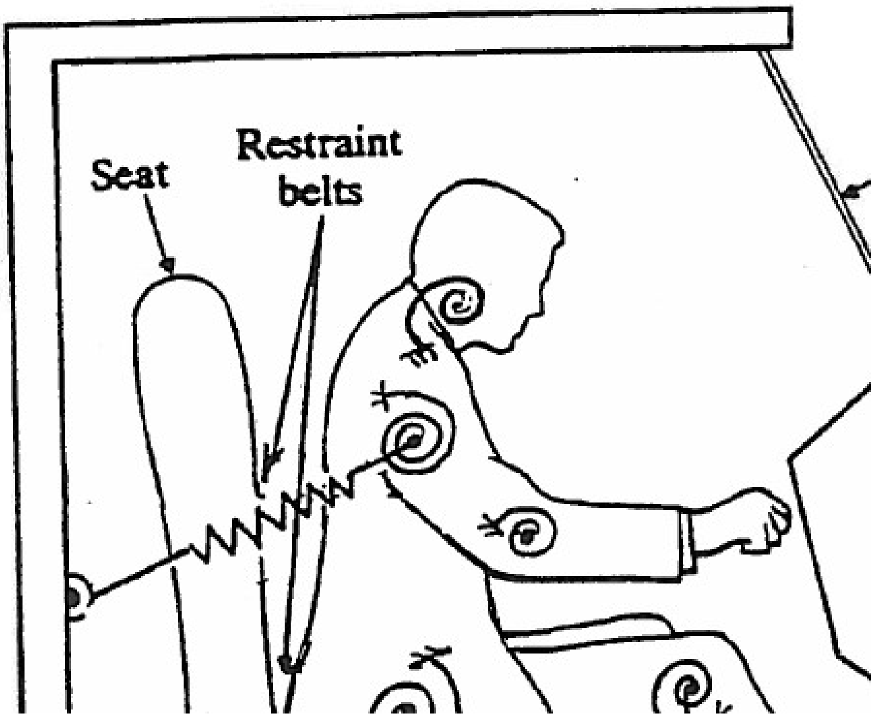
(Fundamental of Vibration)

Issued: Tue. Feb. 20, 2024

Due: Mon. Mar. 04, 2024 (18:00)

1. Rao. P 1.2 Vibration problems formulation

1.2



2. Rao. P 1.9 Spring combinations

1.9 Equivalence of potential energies gives

$$\frac{1}{2} k_{t1}\,\theta^2 + \frac{1}{2} k_{t2}\,\theta^2 + \frac{1}{2} k_1 (\theta\, l_1)^2 + \frac{1}{2} k_2 (\theta\, l_1)^2 + \frac{1}{2} k_3 (\theta\, l_2)^2 = \frac{1}{2} k_{eq}\,\theta^2$$

$$\therefore\ k_{eq} = k_{t1} + k_{t2} + k_1 l_1^2 + k_2 l_1^2 + k_3 l_2^2$$

3. Rao. P 1.10 Spring combinations

1.10 k_{123} = for series springs k_1, k_2 and k_3 :

$$\frac{1}{k_{123}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad ; \quad k_{123} = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}$$

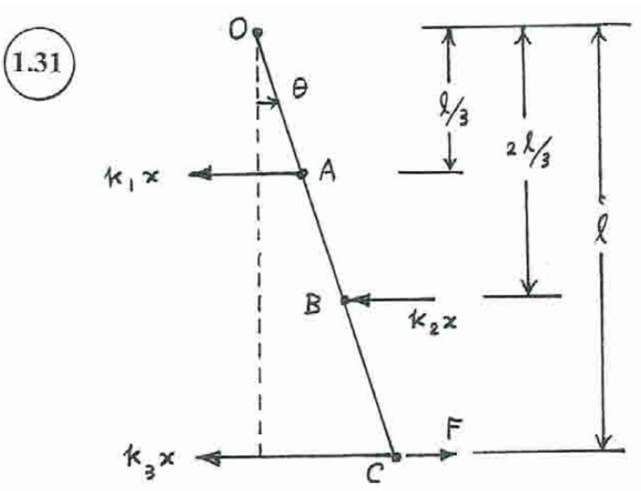
Using energy equivalence,

$$\frac{1}{2} k_{eq} \theta^2 = \frac{1}{2} k_4 \theta^2 + \frac{1}{2} k_{123} \theta^2 + \frac{1}{2} k_5 (\theta R)^2 + \frac{1}{2} k_6 (\theta R)^2$$

$$\therefore k_{eq} = k_4 + k_{123} + R^2 k_5 + R^2 k_6$$

$$= k_4 + \left(\frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1} \right) + R^2 (k_5 + k_6)$$

4. Rao. P. 1.31 Equivalent stiffness



Let the link OABC undergo a small angular displacement θ as shown in above figure. The spring reaction forces are also indicated in the figure. Equilibrium of moments about the pivot point O gives:

$$-k_1 x\left(\frac{\ell}{3}\right) - k_3 x(\ell) - k_2 x\left(\frac{2\ell}{3}\right) + F(\ell) = 0$$

or $$F = \left(\frac{k_1}{3} + \frac{2}{3} k_2 + k_3\right) x \qquad (1)$$

If k_{eq} denotes the equivalent spring constant of the link along the direction of F at point C, we have

$$F = k_{eq}\, x \qquad (2)$$

Equations (1) and (2) give

$$k_{eq} = \frac{k_1}{3} + \frac{2 k_2}{3} + k_3 = \frac{k}{3} + \frac{2}{3}(2k) + (3k)$$

$$\therefore\ k_{eq} = \frac{14}{3} k \qquad (3)$$

5. Rao. P 1.52 Equivalent mass

(1.52) When point A moves by distance $x = x_h$, the walking beam rotates by the angle $\theta_b = \dfrac{x_h}{\ell_3}$.

This corresponds to a linear motion of point B: $x_B = \theta_b\, \ell_2 = \dfrac{x_h\, \ell_2}{\ell_3}$

and the angular rotation of crank can be found from the relation:

$$x_B = r_c \sin\theta_c + \ell_4 \cos\phi = r_c \sin\theta_c + \ell_4 \sqrt{1 - \frac{r_c^2}{\ell_4^2}\sin^2\theta_c}$$

For large values of ℓ_4 compared to r_c and for small values of x and θ_c, we have

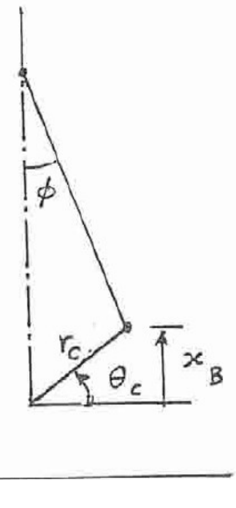
$$x_B \approx r_c \sin\theta_c = r_c\, \theta_c \text{ or } \theta_c = \frac{x_B}{r_c} = \frac{x_h\, \ell_2}{\ell_3\, r_c}$$

The kinetic energy of the system can be expressed as

$$T = \frac{1}{2} m_h \dot{x}_h^2 + \frac{1}{2} J_b \dot{\theta}_b^2 + \frac{1}{2} J_c \dot{\theta}_c^2$$

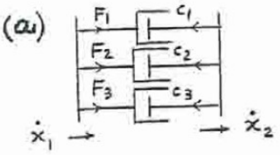
Equating this to $T = \frac{1}{2} m_{eq} \dot{x}^2 = \frac{1}{2} m_{eq} \dot{x}_h^2$, we obtain

$$m_{eq} = m_h + \frac{J_b}{\ell_3^2} + J_c \left(\frac{\ell_2}{\ell_3\, r_c}\right)^2.$$



6. Rao. P.1.55 Equivalent damping

(1.55) (a)



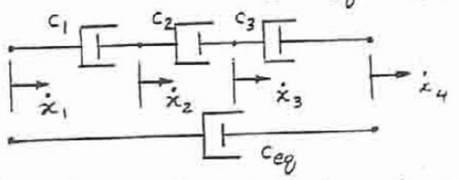
F_i = damping force of $c_i = c_i(\dot{x}_2 - \dot{x}_1)$; $i = 1,2,3$

F_{eq} = damping force of $c_{eq} = c_{eq}(\dot{x}_2 - \dot{x}_1)$

$\equiv F_1 + F_2 + F_3$

$\therefore\ c_{eq} = c_1 + c_2 + c_3$

(b)



$F_1 = c_1(\dot{x}_2 - \dot{x}_1)$

$F_2 = c_2(\dot{x}_3 - \dot{x}_2)$

$F_3 = c_3(\dot{x}_4 - \dot{x}_3)$

$$\dot{x}_4 - \dot{x}_1 = \dot{x}_4 - \dot{x}_3 + \dot{x}_3 - \dot{x}_2 + \dot{x}_2 - \dot{x}_1$$

$$\frac{F_{eq}}{c_{eq}} = \frac{F_3}{c_3} + \frac{F_2}{c_2} + \frac{F_1}{c_1}$$

Since $F_{eq} = F_1 = F_2 = F_3$, $\quad \frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$

(c) Equating the energies dissipated in a cycle,

$$\pi\, c_{eq}\, \omega\, X_1^2 = \pi c_1 \omega X_1^2 + \pi c_2 \omega X_2^2 + \pi c_3 \omega X_3^2$$

where $X_1 = \theta l_1$, $X_2 = \theta l_2$ and $X_3 = \theta l_3$

$$\therefore\ c_{eq} = c_1 + c_2\left(\frac{l_2}{l_1}\right)^2 + c_3\left(\frac{l_3}{l_1}\right)^2$$

(d) Equating the energies dissipated in a cycle,

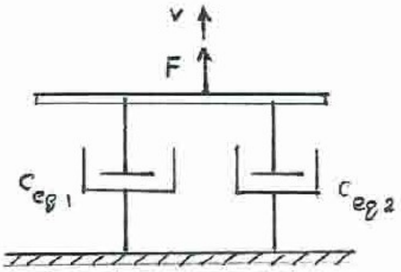
$$\pi\, c_{teq}\, \omega\, \theta_1^2 = \pi\, c_{t1}\, \omega\, \theta_1^2 + \pi\, c_{t2}\, \omega\, \theta_2^2 + \pi\, c_{t3}\, \omega\, \theta_3^2$$

where $\theta_2 = \theta_1\left(\frac{n_1}{n_2}\right)$ and $\theta_3 = \theta_1\left(\frac{n_1}{n_3}\right)$.

$$\therefore\ c_{teq} = c_{t1} + c_{t2}\left(\frac{n_1}{n_2}\right)^2 + c_{t3}\left(\frac{n_1}{n_3}\right)^2.$$

7. Rao. P.1.74 Equivalent damping

(1.74)



For two dampers in series, the equivalent damping constant is given by

$$\frac{1}{c_{eq_1}} = \frac{1}{c_1} + \frac{1}{c_1} = \frac{2}{c_1}$$

or $$c_{eq_1} = \frac{c_1}{2}$$

For two dampers in parallel, the equivalent damping constant is given by

$$c_{eq_2} = c_2 + c_2 = 2\,c_2$$

Thus the system can be replaced by the two equivalent dampers in parallel as shown in the figure above.

The overall equivalent damping constant is given by

$$c_{eq} = c_{eq_1} + c_{eq_2} = \frac{c_1}{2} + 2\,c_2$$

so that

$$F = c_{eq}\, v$$