國立成功大學機械工程學系

機械振動學(Mechanical Vibrations)

Spring Term 2023

QUIZ II

May 24 2023 (Wednesday)

6:30 – 8:30 PM (可延長至 8:45)PM

RM. 91302

Note:

Problem I: Close Book/Close Notes

Rest Problems: Close Book but a A4 sheet of notes is permissible (請不要在 handout 上面黏貼其它的資料)

先做第一題. 第一題交卷後, 可拿出預先準備的 Notes Sheet, 做其他的題目.

共 7 大題 + 1 Bonus. 本次題目 (含此頁) 計 6 頁

Total 128 + 10 Bonus Points

Part I. Closed books/notes (9 Questions, 50 Pts)

Problem 1. Please briefly answer the following questions (Closed books/notes)

A. (5 Pts) In section 4.6 (Response Spectrum), we have utilized Fig. 4.15 (see below) to illustrate how to plot the response spectrum for a system subjected to a given forcing manner. Now, please try your best to explain how to plot the response spectrum.

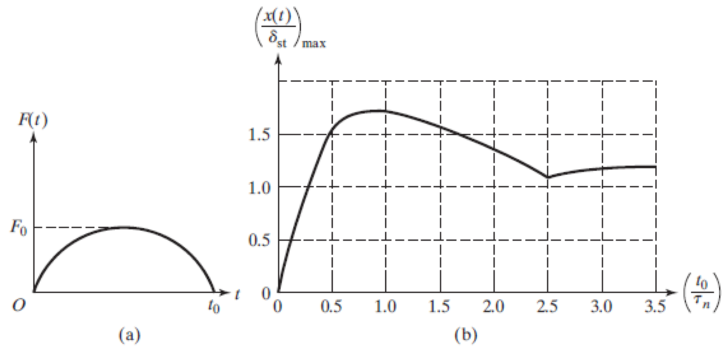


FIGURE 4.15 Response spectrum due to a sinusoidal pulse.

B. (5 Pts) For a SDOF system subjected to a general non-periodic loading shown below, the general solution can be expressed as:

$$x(t) = \frac{1}{m\omega_d}\int_0^t F(\tau)\, e^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t - \tau)\, d\tau$$

Please try your best to tell us the physics/math of the above equation and to explain why the equation is the general solution of the vibration problem.

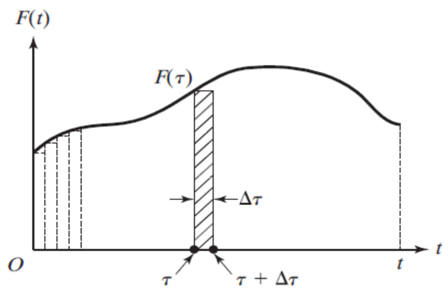


FIGURE 4.9 An arbitrary (nonperiodic) forcing function.

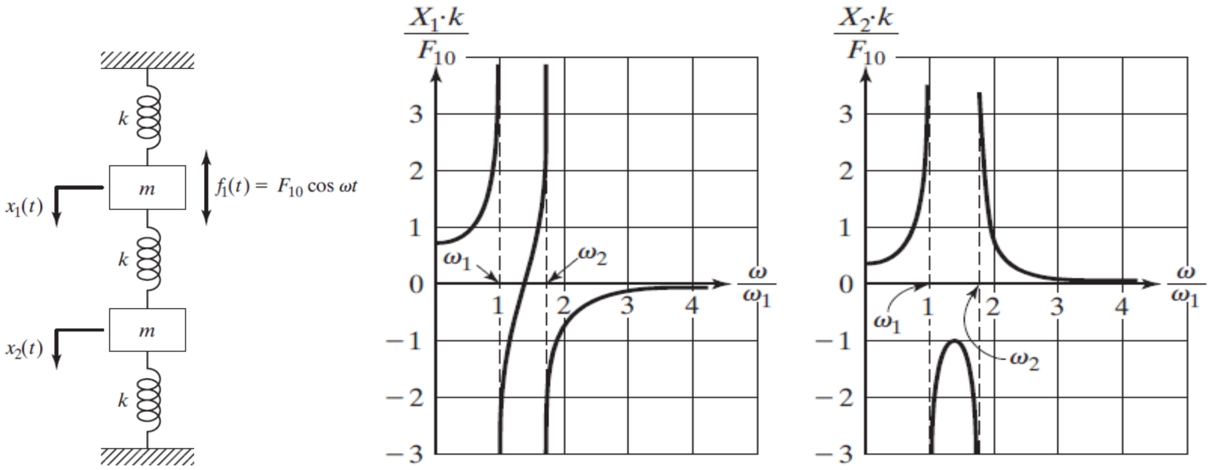
C. (5 Pts) In textbook, velocity spectrum (i.e., S_v), displacement spectrum (i.e., S_d), and acceleration spectrum (i.e., S_a) are also introduced. From a mechanical system designer's perspective, what are the importance of these spectra?

D. (5 Pts) In MDOF systems, why [K] matrix is symmetric ?

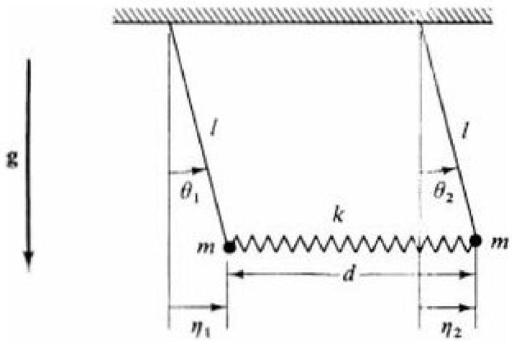
E. (5 Pts) What are principal coordinates and mode shapes? Please try to explain it by mathematics and physics/engineering.

F. (5 Pts) What happen if a vibration system contains eigenvalues of value = 0? And what happen if the system has repeated eigenvalues?

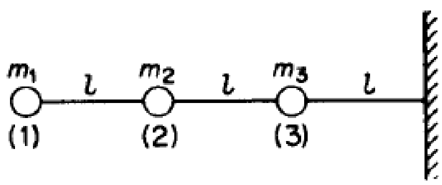
G. (5 Pts) Consider a 2-DOF system shown below, its frequency respose function plots are also shown. Please estimate the values of its resonace points and anti-resonance points. (given: m = 2 Kg; k= 100N/m).



H. (5 Pts) The following figure shows a simple coupled pendulum. Please use any methods to find its natural frequencies and corresponding mode shapes.



I. (10 Pts) In Ch. 6, we spent almost 1 hour to explain stiffness influence coefficient. Now, consider a 3 DOF system shown below. The degree of freedoms are the vertical vibrations of the three masses. These masses are connected by three beams. Please schematically tell us how to find the stiffness matrix of the vibration system. You may need to draw the necessary shapes.



Part II. A sheet of notes is permissible (6 Problems + Bonus, 78 Pts + 10 Pts)

Problem 2. Command Shaping Technologies (12 Pts)

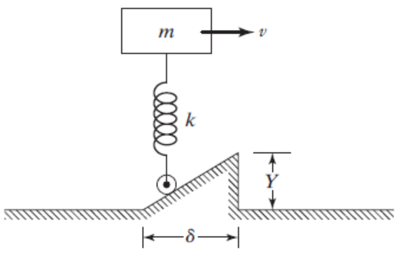
Consider an undamped SDOF vibration system with mass **m=1 Kg**, stiffness **k = 4π² N/m** subjected to an excitation force ***F(t)***. Where $F(t)=\begin{Bmatrix} 0 & t\le 0 \\ 1 & t>0 \end{Bmatrix}$. Both the initial displacement and velocity are zero.

(a) Please find the response of the vibration system. You can either write the mathematics or sketch the behavior with sufficient information. (5 Pts)

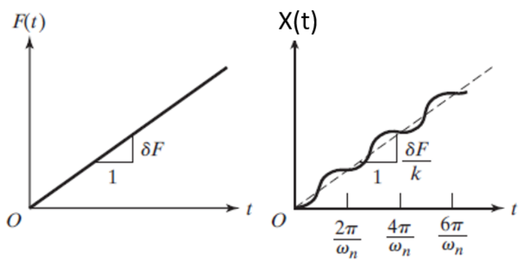
(b) Now, instead of exciting the system with the original *F(t)*, the same vibration system is subjected to a new excitation ***G(t)***. Where $G(t)=\begin{Bmatrix} 0 & t\le 0 \\ 2 & 0<t\le 0.5 \\ 4 & t>0.5 \end{Bmatrix}$. Please find the response of the system. Again, you can either write the mathematical expressions or provide a sketch with sufficient information. (7 Pts)

Problem 3. Convolution Integrals (12 Pts)

A vehicle traveling at a constant speed ***v*** in the horizontal direction encounters a triangular road bump, as shown in the following figure. Treating the vehicle as an undamped spring-mass system, determine the response of the vehicle in the vertical direction.



The following information in your textbook might be useful for solving the problem: Consider a SDOF vibration system subjected to a ramp input $m\ddot{x}+kx=F(t)$, its steady state solution X(t) can be expressed as $x(t) = \frac{\delta F}{\omega_n k}[\omega_n t - \sin \omega_n t]$



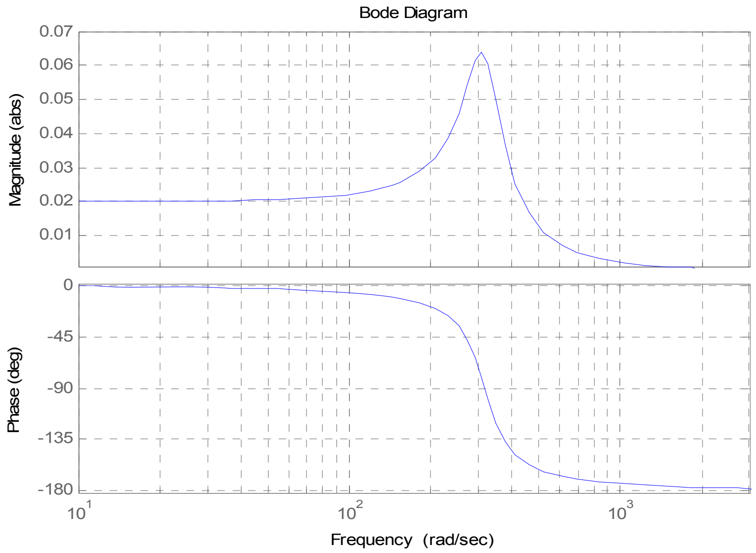
Problem 4. **System subjected to Multiple Inputs** (13 Pts)

The frequency response function (or Bode Plot) for a vibration system (i.e., ***X(s)/F(s)***) is shown below. Please try to answer the following questions. 注意：為了有效估計, magnitude plot 之縱軸目前是以線性 scale (而非 Log or dB scales)

(a) Please estimate the static sensitivity of the system? (3 Pts)

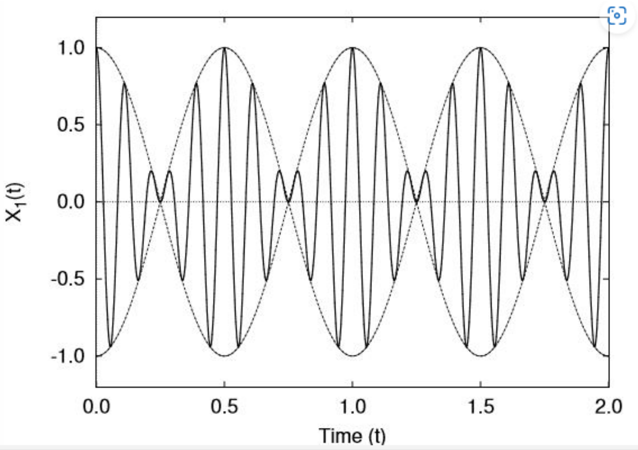
(b) Please estimate the natural frequencies of the system? (3 Pts)

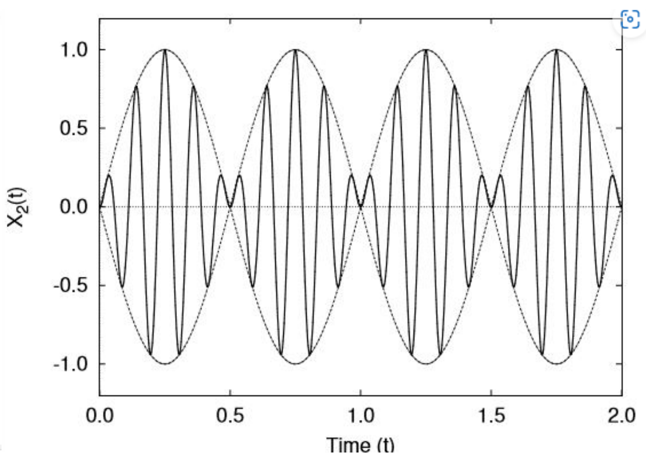
(c) If the input signal is $F_{in}(t) = 2\sin(30t + 30°) + 3\cos(150t - 20°) + 20\cos(400t + 70°)$. Please find the corresponded ***X(t)***. (7 Pts)



Problem 5. **Natural frequencies of 2-DOF System** (10 Pts)

The following figures are the free response of a 2-DOF vibration system. Please estimate the natural frequencies of the system.



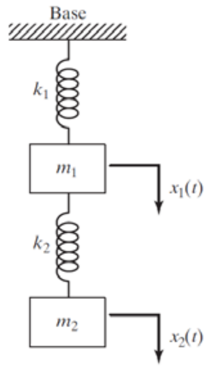


Problem 6. **Find Forced Responses of 2-DOF System** (15 Pts)

Consider the system shown below, where $\mathbf{m_1=m}$, $\mathbf{m_2=2m}$, $\mathbf{k_1=k}$, and $\mathbf{k_2=2k}$. Gravity effect is **not** considered in this problem.

(a) Please find the equation of motion (5 Pts)

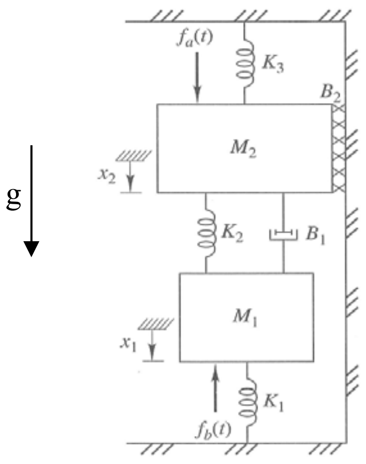
(b) Please find the natural frequencies and the corresponding mode shapes. (10 Pts)

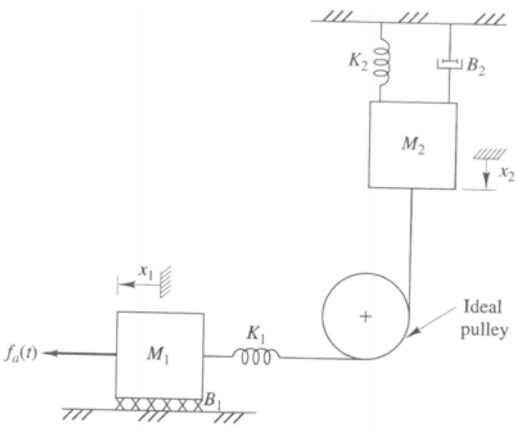


Problem 7. **Modeling of 2-DOF System** (16 Pts)

(a) Please find the equation of motion for the 2-DOF system shown in (Fig. 7a). The friction damper $\mathbf{B_2}$ can be treated as a viscous damper between $\mathbf{M_2}$ and the fixed reference. (8 Pts)

(b) Consider the system shown in (Fig. 7b), please find the equation of motions. (8 Pts)





Bonus (10 Pts)

Consider the three-coupled pendulum problem shown below, please determine (and plot) its vibration modes and natural frequencies.